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IN LIVING MEMORY: THE DYNAMICS OF EVENT RECOLLECTION IN A STABLE POPULATION

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ABSTRACT

We model a stable population that has experienced an important historical event and the declining proportion of the population that remembers that event, as time passes. The proportion is determined by the demographic characteristics of the population, including its age distribution, the natural rate of growth, the underlying birth rate, the life table probabilities to which the population is subject, and the effects of immigration and emigration under alternative assumptions about the nature of the event. (We distinguish between “local” and “universal” events.) It is determined also by the choice of an age of awareness of children at the time the event occurred. We preface development of the model by noting examples of major events of the kind we have in mind and, after development, explore the model’s sensitivity to different parameter specifications, by experimental simulation. The output of each experiment is a sequence of “remembering” proportions at successive decade intervals and the corresponding mean ages of the “rememberers” in relation to the overall mean age of the population.

JEL Categories:

C63: Computational Techniques • Simulation Modeling

J1: Demographic Economics

J10: General

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Keywords/phrases

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1. INTRODUCTION

The word memory has particular interpretations and shades of meaning – in psychology and neuroscience, of course, but in other disciplines and areas of study as well. One speaks of personal memory, collective memory, historical memory, social memory, cultural memory, community memory, computer memory, virtual memory, working memory, living memory. The present paper is about what we may call population memory, or perhaps collective living memory. It is about the proportion surviving of a population that was living and aware of an important historical event when it occurred, and the demographic determinants of that proportion as time moves on.

“How many people are old enough to remember?” is a question of common popular interest: How many remember the assassination of President John F. Kennedy, the first landing on the moon, the coming down of the Berlin wall? On September 11, 2015, The Washington Post published an article headed, “A quarter of Americans are too young to remember 9/11” (Bump, 2015). World War II looms large in the history of the 20th Century but the number who actually remember the war is small and dwindling. Life table probabilities are taking their toll.

The factors that determine the population proportion of surviving “rememberers” of an event are in a general sense rather obvious. The purpose of this paper is to coordinate them in the form of a model, to use the model to explore their relative importance, and to simulate the paths of the surviving proportion in elapsed time, under alternative assumptions. Every country has its own demographic characteristics and history: mortality rates, fertility rates, migration rates, age distributions, life expectancies all differ internationally and change through time. To

provide a model for the population of any particular country (or other geographic area – state, province, city) would obviously have to take that into account. Our aim here is rather to provide a general framework for thinking about the survival issue and to conduct illustrative experiments. To that end we specify the population of interest to be stable and model it accordingly.

Our focus then is population memory. We are not aware of previous studies that bear directly on the present paper (readers may be able to correct us). However, brief comment on some other descriptive terms that one sees attached to the word memory may help provide context, in particular historical, social, and collective memory. All represent, as areas of interest, past societal behaviour, relationships, and institutions. The terms are sometimes used as meaning essentially the same thing, sometimes as distinct. Writing in the International Encyclopedia of Political Science, Katherine Hite (2011) offers the following: “The concept of ‘historical memory,’ often expressed as ‘collective memory,’ ‘social memory,’ . . . refers to the ways in which groups, collectivities, and nations construct and identify with particular narratives about historical periods or events.” From that perspective, population memory, as we define it, might be regarded as an explanatory consideration: the prevalence or scarcity and age of people who today remember the past occurrence of an event might help to explain current societal attitudes in some circumstances under the heading of historical memory. But memory, in our setting, population memory, covers less than a century, the time it takes for all individuals alive and aware of an event when it occurred to die off.

2. HISTORICAL EVENT

What we term an historical event could in principle be of any kind – a war, natural disaster, major economic disturbance, political or societal disruption of some sort – as long as it is commonly viewed as important by the members of a population and triggers lasting memories among those who are old enough to have been aware of it when it happened. By way of setting the scene we provide in Table 1 the list of what Gallup News Service called “the most important events of the century from the point of view of the people”. The century is the 20th and the list is taken from a news release by Gallup on December 6, 1999, the result of a random poll. There are 18 events listed in the table, in order of importance as determined by Gallup from the polling responses. (See Newport et al., 1999, for information about the conduct of the poll and discussion of results.)

The 20th century list of events in Table 1 was compiled by surveying respondents in the United States and reflects American views and experiences. (Some of those polled would have had personal memories of an event, others would not have been living when it occurred.) Events in other contexts might include, as examples, the Halifax Canada explosion of 1917, the Mexico earthquake of 1985 and, going into the 21st century, the 9/11 terrorist attacks of 2001, the Haiti earth quake of 2010, and November 7, 2016, the date on which Donald Trump became President Elect of the United States. But the aim here is not to catalogue such events, just to note some as examples of what we have in mind in what follows.

We assume for present purposes that events can be classified by geographic boundaries. The population of interest – the population for which an event is considered

especially important by its members -- might be a national population or the population of a city, province, or state. This is particularly relevant to the treatment of immigration. An event that may be of great importance to the population in one geographic area but of little or no interest outside of that area we will call a *local* event. An event that is of more widely spread interest we will call a *universal* event. The latter term may apply in particular to countries or other areas from which immigrants come. The point is that immigrants may arrive already having significant personal memories of an event if it is classified as universal (World War II, for example) but no such memories if it is local (the opening of the Toronto subway system, say). To take the most obvious contemporary example of a universal event, the world-wide covid-19 pandemic is raging as we write. (What proportion of the world's population will remember it in 50 years?)

An event may occur at a precise time (the assassination of President Kennedy, the 9/11 terrorist attacks) although its consequences may last much longer (the passage of legislation in 1918 granting Canadian women the right to vote in federal elections; see Elections Canada, 2020). Alternatively it may go on for several years (World Wars I, II). In the theoretical model below we will treat an event as if it occurred at a point in time (represented in the model as the start of a particular year) but longer-lasting events could be dealt with by choosing the starting point, the end point, or a midpoint, or perhaps by repeating the calculations for both starting and end points, if that seemed appropriate. Multiperiod events of particular interest that might be treated in this way in economics, for example, could include the Great Depression, defined as lasting from 1929 to 1938 or 1939 (Amadeo, 2020, Pells and Romer, 2019) or in the United States, with an extended timeline based on continuing unemployment, until 1941 when the

U.S. entered World War II (Amadeo, 2020). A more recent example is the Great Recession of 2007 – 2009. (Precise definitions of business cycle turning points are provided by the National Bureau of Economic Research for the United States; see NBER, 2020.) Another example, from a different perspective, would be the Spanish Flu epidemic of 1918 – 1920 (Taubenberger and Morens, 2006).

3. THE MODEL WHEN THE POPULATION IS CLOSED

Consider a country with population of size N_t in year t . (We say country but it could be any geographic area of interest.) Population counts, births, and deaths occur at the beginning of the year. The population is closed (no migration in or out) and is dynamically stable. It has constant annual natural rate of growth $g \geq 0, < 0$ per year, constant fertility and survival rates, a constant sex ratio at birth, and is distributed among single years of age by vector $n_t = (n_{jt}; j = 0, 1, \dots, h)$, 0 being the age at birth and h the oldest age at which there are any survivors. Stability implies that all age groups are increasing at the same rate and hence that the proportionate distribution is unchanging. Let B_t be the number surviving to age j ($s_0 = 1$, $s_{h+1} = 0$). Noting that the population of age j in year t is equal to $B_t(1 + g)^{-j}s_j$ we may write

$$N_t = (1 + g)^t N_0 = \sum_{j=0}^h (1 + g)^t n_{j0} = B_t \sum_{j=0}^h (1 + g)^{-j} s_j$$

and then express the birth rate as

$$b = B_t/N_t = (\sum_{j=0}^h (1 + g)^{-j} s_j)^{-1}$$

thus making explicit the connection between the birth rate and the natural rate of growth. (The derivation is consistent with the discrete form of the Euler–Lotka equation; see Keyfitz, 1968, Chapter 7, Kot, 2001, Chapter 20.)

Now assume the occurrence of an historical event E at the beginning of year $t = 0$. The population at the time of occurrence is N_0 and the population k years later is $N_k = (1 + g)^k N_0$. The youngest age at which a child could be aware of the event is assumed to be a so that only the population at ages a and older will have been aware of it at $t = 0$ and only their survivors at ages $a + k$ and older will remember the event k years later. (We abstract from loss of memory due to neurological conditions.) The size of the overall population is changing with time at rate g while the “remembering” portion of the population, R let us call it, is decreasing as a consequence of mortality rates that rise with age. A result of interest is then the ratio r of one to the other. For convenience set $N_0 = 1$, here and below, thus making $B_0 = b$. Then r , when k years have elapsed since the event, can be derived as follows:

$$N_k = b \sum_{j=0}^h (1 + g)^{k-j} s_j; \quad R_k = b \sum_{j=a+k}^h (1 + g)^{k-j} s_j; \quad r_k = R_k / N_k$$

A cohort in a stable population with a set S of survival rates s_j ($j = 0, 1, \dots, h$) has life expectancy at birth e_0 and mean age $\bar{x}$, calculated as

$$e_0 = \sum_{j=0}^h s_j; \quad \bar{x} = (\sum_{j=0}^h j s_j) (\sum_{j=0}^h s_j)^{-1} = (\sum_{j=0}^h j s_j) e_0^{-1}$$

The mean age for the population as a whole (calculated at $t = 0$ but the same for all years) is

$$\bar{y} = (\sum_{j=0}^h j (1 + g)^{-j} s_j) (\sum_{j=0}^h (1 + g)^{-j} s_j)^{-1}$$

which reduces to $\bar{x}$ for $g = 0$. The mean age of the remembering population in year k is

$$\bar{y}_{rk} = (\sum_{j=a+k}^h j(1+g)^{k-j} s_j) (\sum_{j=a+k}^h (1+g)^{k-j} s_j)^{-1}$$

4. THE MODEL WHEN THE POPULATION IS OPEN

Now introduce migration, denoting annual totals by M for immigrants, X for emigrants, and their age components by vectors m and x . (Migration, like births and deaths, occurs at the beginning of a year for all ages (including $j = 0$, newborn children). Assume that migrants have the same vital rates as the rest of the population (fertility, survival, sex ratio at birth). Assume too, in the further interest of simplification, that they also have the same proportionate age distributions at the time of migration. Thus, for any year t , $m_{jt}/M_t = x_{jt}/X_t = n_{jt}/N_t$; $j = 0, 1, \dots, h$. Let the proportions of immigrants entering the population and emigrants leaving in any year t be $\beta = M_t/N_{t-1}$ and $\gamma = X_t/N_{t-1}$, which can be interpreted as an addition to and a deduction from the natural rate of growth. Thus the population retains its stability and the annual overall rate of growth is now $g + \beta - \gamma$. Note then that an increase in the population of age j by immigration is exactly equivalent in numerical effect to an increase in births j years earlier, adjusted for survival, and similarly for a decrease by emigration. Again setting $N_0 = 1$, the population k years after E can be calculated as

$$N_k = b^* \sum_{j=0}^h (1+g+\beta-\gamma)^{k-j} s_j$$

where b^* is the migration-equivalent birth rate obtained by replacing $(1+g)$ with $(1+g+\beta-\gamma)$ in the earlier calculation of b .

The remembering population, like the general population, will be depleted by emigration but the effect of immigration will depend on the nature of the event E . If E is local, immigration will have no effect on the remembering population but if it is universal immigrants will enter the population having been aware of the event in their country of origin if they were of age a or older when it occurred, at $t = 0$, and thus they will enter the remembering part of the population when they arrive, at $t \geq 0$. (A strict interpretation of “universal” is more general than necessary; all that is necessary is that immigrants had been aware of the event in their particular areas of origin where the event was recognized and considered important.) Thus the remembering population at $t = 0$, “the moment of the event”, is

$$R_0 = b^* \sum_{j=a}^h (1 + g + \beta - \gamma)^{-j} s_j$$

and the remembering population k years later consists of the survivors of R_0 depleted by k years of emigration if E is local:

$$R_k = (b^* \sum_{j=a+k}^h (1 + g + \beta - \gamma)^{k-j} s_j) (1 - \gamma)^k \quad (E \text{ local})$$

If E is universal the survivors of R_0 are also augmented by k years of immigration, converting the migration adjustment from gross to net immigration:

$$R_k = (b^* \sum_{j=a+k}^h (1 + g + \beta - \gamma)^{k-j} s_j) (1 + \beta - \gamma)^k \quad (E \text{ universal})$$

The ratio to the overall population is then calculated as before: $r_k = R_k / N_k$. The survival rates are the same as those for a closed population and so e_0 and $\bar{x}$ are unchanged by migration. The overall population mean age is given by

$$\bar{y} = (\sum_{j=0}^h j(1 + g + \beta - \gamma)^{-j} s_j) (\sum_{j=0}^h (1 + g + \beta - \gamma)^{-j} s_j)^{-1}$$

If E is either local or universal (it makes no difference) the mean of the remembering population is given by

$$\bar{y}_{rk} = (\sum_{j=a+k}^h j(1 + g + \beta - \gamma)^{k-j} s_j) (\sum_{j=a+k}^h (1 + g + \beta - \gamma)^{k-j} s_j)^{-1}$$

(Assuming the same age distribution for migrants as for non-migrants means that migration does not alter the age distribution of rememberers.)

This sets the stage for the experimental calculations below with alternative assumptions about the parameters of the model.

5. ALTERNATIVE PARAMETER SETTINGS

The parameters include the youngest remembering age a , the natural rate of growth g (or equivalently, the birth rate b), the set of survival rates S , the rate of immigration β , the rate of emigration γ , and we may add to the list the local/universal classification of the event E which can be thought of as a (0,1) parameter choice (0 for local, 1 for universal). Our aim is to illustrate the role and importance of each in determining the time path of the proportion r of the population that remembers E as the decades pass since its occurrence. To that end we choose a few alternatives for each type of parameter and note, for each type, the one we have selected as a standard value.

Youngest remembering age

$a = 8, 12, 16, 20$. The standard is $a = 12$. Note that 8 is the oldest year of Early Childhood in Erikson's stages of psychosocial development (Erikson, 1950, Syed and McLean, 2017); 12 is the oldest year of Middle Childhood; 16 is the middle year of Adolescence; and 20 is the first year of Early Adulthood. (Age 20 can also be viewed as a rough approximation to national voting age, which is most commonly 18 but varies among countries from 16 to 25; see World Atlas, 2017.) The idea here is that the significance of an event for a young person may differ with age, depending on the type of event. (A war may be immediately significant to a relatively young child while the implications of an important piece of legislation or a major judicial decision may require more maturity.)

Natural growth rate (%)

$g = -1.0, 0.0, 1.0, 2.0$. The standard is $g = 0.0$.

Survival rates

$S = S_0, S_1, S_2, S_3$. The standard, S_0 , is based on Canadian life table l_x values for the three-year period 2015–2017. (The table is commonly referred to as the 2016 life table; see Statistics Canada, 2019.) S_0 represents approximately the middle of the top (fourth) quartile group of survival rates ranked internationally according to life expectancy (World Population Review, 2020) and provides a convenient base line for purposes of the calculations. The maximum age in the Canadian life table is 110 and h is set accordingly. S_1, S_2 , and S_3 are obtained by shifting the S_0 rates from older to younger ages (to the left and downward, from a graphical point of

view) so as to represent, in a rough way, situations (countries or other areas) with lower life expectancies: a shift of v years produces an initial shifted survival rate $\bar{s}_j$ equal to the original rate s_{j+v} and a new maximum age $h = 110 - v$. A proportional adjustment (very small) is made to all of the initial shifted rates to make the survival rate at age 0 equal to 1, as required by the model, thus obtaining the final new s_j rates. The four survival functions are plotted in Figure 1 and the corresponding values of e_0 and $\bar{x}$ are shown in Table 2, discussed below. The shifted distribution S_1 is approximately at the middle of the third quartile group of countries according to the international rankings of life expectancies, S_2 is approximately at the middle of the second quartile group, S_3 is approximately at the middle of the first quartile group. The survival rate shifts associated with S_1, S_2, S_3 are $v = 6, 10, 18$ years, respectively. We emphasize that we do not interpret S_1, S_2, S_3 (and the shifting procedure on which they are based) as carefully derived representations of actual country survival rates, merely as convenient approximative devices for exploring the effects of shifting the rates away from S_0 .

Immigration rate (%)

$\beta = 0.0, 0.5, 1.0$. The standard is $\beta = 0.0$.

Emigration rate (%)

$\gamma = 0.0, 0.5$. The standard is $\gamma = 0.0$.

Classification of event

E is either local or universal. Local is the standard. Note that if $\beta = 0$ (no immigration) universality has no effect: results are the same whether E is classified as universal or local.

6. EXPERIMENTAL CALCULATIONS

Calculations of r using the foregoing parameter choices are shown in Table 2 at 10-year intervals: $k = 0, 10, \dots, 80$. Results based on the standard set are shown in the first row and those based on alternative parameter choices are shown in the five groups below that, one or two types of parameters in each group being allowed to depart from standard values in each set of calculations. For convenience in interpretation the standard row is repeated in some cases in the table in order to place results based on a standard parameter choice in the context of the overall set of alternatives for that parameter. (Example: $g = 0$, its standard value, is repeated in the context of alternative values of g .) Also shown in the table, at the bottom, are results for the combinations of parameters (among the ones we have specified) that yield the highest and lowest values of r . (The youngest remembering age a is held at 12 in those calculations while the others are allowed to vary.)

The dominant feature of the results shown in the Table 2 is that r declines monotonically with elapsed time k , whatever the parameter configuration. The principal issue of course is how the speed of decline varies with alternative choices. Considering first the results based on the standard parameter set, 85.5 percent of the population are aware of an event when it occurs, the other 14.5 percent being too young to appreciate the event's significance. Twenty years later 61.5 percent remember the event, at 40 years the proportion is down to 37.8 percent, and at 60 it is down to 15.8, roughly one in six. By 80 years after the event almost nobody survives (proportionately) to say to the grandchildren, "I remember when that happened".

The youngest age of awareness matters significantly. At $a = 12$ the initially aware proportion r is 85.5 percent. Increasing a by four years, from 12 to 16, reduces r by roughly five percentage points and increasing it to 20 (young adulthood) reduces r by another five; all future proportions, $k > 0$, are affected accordingly. At the other end, lowering the age to 8 raises the initially aware fraction by about five percentage points, to a little over 90 percent, with corresponding longer-term effects. Which age might be thought appropriate would depend on the nature of the event being considered – and of course, we are dealing with a theoretically stable population; the effects on r in an actual population might be somewhat different. We assume $a = 12$, the standard assumption, in all of the remaining calculations in Table 2.

In the absence of migration (the closed form of the model) the natural rate of growth g has a major effect on both the initial age distribution of the stable population and the path of r as k increases. On the one hand it determines the proportions of the population under and within the remembering age range; on the other it determines the rate at which the total population -- the denominator in the calculation of r -- is increasing, while having no effect on the remembering population in the numerator, which depends only on survival rates. The choices for experiment in Table 2 range from negative growth of 1.0 percent per year to positive growth of 2.0 percent (and an associated birth rate from 8 per thousand to 25 per thousand). Going from population decline to strong positive growth brings about a major shift in the initial awareness proportion, moving it from 90 percent of the total population down to a little under 75 percent. The subsequent results include values of r of 71.4, 48.6, and 22.7 percent at $k = 20, 40$, and 60 respectively, when $g = -1.0$ percent, compared with 41.3, 19.9,

and 6.5 when g is 2.0. In short, rapid growth implies a faster declining remembering proportion.

Changes in survival rates affect both the numerator and denominator of r but the effect on the numerator is proportionately greater: the longer is mean life expectancy (and associated mean cohort age) the greater is the proportion of the remembering population, whose continuing numbers depend only on survival. This is evident from the results of shifting the survival function downward and away from S_0 , as described above. The effects on the initial age distribution are relatively small; greater effects show up as time passes (k increases). By 40 years after an event r ranges from 37.8 percent (at longest life expectancy) down to 22.7 (at shortest expectancy). By 60 the range is 15.8 down to 2.7).

Once migration is introduced (in the lower half of Table 2) the local vs. universal classification of an event comes into play. Four immigration/emigration scenarios are defined and repeated, first with E local, then with E universal. With E local immigration affects the total population but not the remembering part of it. Emigration on the other hand affects both: rememberers leave at the same rate as everyone else. Overall though, at the rates we have specified, introducing migration has only minor effects on r when the event is classified as local. Reclassifying it as universal increases r in all cases as immigrants augment the remembering population but the effects are still moderate. (Specifying higher rates of immigration, especially net immigration, would of course have greater effects.) Note that when immigration and emigration are both positive but net immigration is zero ($\beta = \gamma$) the results are the same as

when there is no migration: the values of r are identical to those with standard parameters since vital characteristics are the same for immigrants and non-immigrants in the model.

The final calculations in the table maximize and minimize the r sequence over the full set of our chosen alternative parameter values (aside from a , which is held at age 12). The maximum is obtained with the highest survival rates (S_0), the lowest natural growth rate (-1.0%), and no migration in either direction. (E is classified here as local in the table but with no immigration the classification does not matter.) The minimum is obtained with the lowest set of survival rates (S_3), the highest natural growth rate (2.0%), the highest immigration rate (1.0%), no emigration, and E local. The minimum r time path is plotted in Figure 2, along with the time paths for the four alternative growth rates shown in Table 2.

As r declines with elapsed time k the surviving rememberers' mean age $\bar{y}_r$ increases. Table 3 shows the effects. The format is similar to Table 2, with $\bar{y}_r$ replacing r . The mean age of the total population $\bar{y}$, which is independent of k , is shown in brackets for each parameter choice.

A rough generalization is that the mean age of rememberers rises by five to seven years with the passing of every decade since an event. The evolving age pattern varies with the parameter assumptions of course; the minimum r case is the most notable variant in the table and the S_3 survival-rates case is next in line. Based on standard parameter values, though, the average age is about 54 after 10 years, 59 after 20, 64 after 30, and 70 after 40 – or, to put it differently, the rememberers are roughly 12, 17, 22, and 28 years older than the general population, on average, at those points in elapsed time.

7. SUMMING UP

We have assumed a stable population in this paper and constructed a model to explore the effects of differences in demographic characteristics on the declining proportion that has living memory of an historical event, as time passes. There are several factors that bear on the time path of that proportion. The age at which a child can be expected to be aware of such an event affects the initial level, and hence all subsequent ones. The natural rate of growth of the population and the underlying birth rate affect the age distribution of the general population and hence both the initial awareness proportion and the initial age distribution of rememberers. The set of life table probabilities to which the population is subject has obvious implications for the survival rates of rememberers, as well as the general population (which represents the denominator in calculating the proportion). An important consideration here is that while the general population responds to differences in the natural growth rate the rememberers are unaffected by natural growth after an event has occurred. Migration rates bear directly on the growth of the general population but how they affect the remembering population may depend on the nature of the historical event. If the event is what we call local then immigrants will not enter the remembering population but emigrants may leave it. If instead the event is classified as universal, then immigrants will already be aware of it when they arrive, and hence go into the remembering population as well as the general population. We have tried to capture all of these effects in our model and to display them by way of simulations under alternative assumptions about the model's parameters, showing the differences in time paths of the remembering proportion and the associated differences in average age.

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Figure 1. Alternative Survival Functions

Figure 2. The Remembering Population as Percent of Total Population (r) by Years Since an Event, Selected Alternative Parameter Specifications

Table 1: 'The Most Important Events of the Century' According to a Gallup Poll

	MI	I	SI	NI	DK
	%	%	%	%	%
1. World War II	71	21	5	2	1
2. Women gaining the right to vote in 1920	66	20	11	3	0
3. Dropping the atomic bomb on Hiroshima in 1945	66	20	9	4	1
4. The Nazi Holocaust during World War II	65	20	9	5	1
5. Passage of the 1964 Civil Rights Act	58	26	13	2	1
6. World War I	53	28	11	5	3
7. Landing a man on the moon in 1969	50	30	15	5	*
8. The assassination of President Kennedy in 1963	50	29	15	5	1
9. The fall of the Berlin Wall in 1989	48	30	19	3	0
10. The U.S. Depression in the 1930s	48	29	18	3	2
11. The breakup of the Soviet Union in the early 1990s	46	31	19	3	1
12. The Vietnam War in the 1960s and early 1970s	37	31	20	11	1
13. Charles Lindbergh's transatlantic flight in 1927	27	33	28	11	1
14. The launching of the Russian Sputnik satellites in the 1950s	25	34	28	12	1
15. The Korean War in the early 1950s	21	36	32	10	1
16. The Persian Gulf War in 1991	18	38	32	11	1
17. The Impeachment of President Bill Clinton in 1998	15	25	24	35	1
18. The Watergate scandal involving Richard Nixon in the 1970s	14	32	34	19	1

MI = Most important, I = Important, SI = Somewhat important, NI = Not important, DK = Don't know

* = Less than 0.5%

Note: This table is reproduced from polling results released by Gallup Inc on December 6, 1999. See Newpork et al. (1999) for information about the conduct of the poll and discussion of results.

Table 2: The Remembering Population as Percent of Total Population ($r = R/N$) k Years After an Event E

Parameter specification	$k = 0$	$k = 10$	$k = 20$	$k = 30$	$k = 40$	$k = 50$	$k = 60$	$k = 70$	$k = 80$
Standard values	85.5	73.5	61.5	49.6	37.8	26.4	15.8	6.8	1.4
Youngest remembering age									
$a = 8$	90.3	78.3	66.3	54.3	42.5	30.9	19.9	10.1	3.1
$a = 12$	85.5	73.5	61.5	49.6	37.8	26.4	15.8	6.8	1.4
$a = 16$	80.7	68.7	56.7	44.9	33.2	22.0	11.9	4.1	0.5
$a = 20$	75.9	63.9	52.0	40.2	28.7	17.8	8.4	2.1	0.1
Alternative natural growth rates									
$g = -1.0\%$ ($b = 8$ per 1000)	90.3	81.3	71.4	60.5	48.6	35.8	22.7	10.5	2.3
$g = 0.0\%$ ($b = 12$ per 1000)	85.5	73.5	61.5	49.6	37.8	26.4	15.8	6.8	1.4
$g = 1.0\%$ ($b = 18$ per 1000)	79.8	64.7	51.2	39.0	28.1	18.5	10.4	4.2	0.8
$g = 2.0\%$ ($b = 25$ per 1000)	73.5	55.8	41.3	29.5	19.9	12.3	6.5	2.5	0.4
Alternative survival rates									
S_0 ($e_0 = 82.5$, $\bar{x} = 42.1$)	85.5	73.5	61.5	49.6	37.8	26.4	15.8	6.8	1.4
S_1 ($e_0 = 76.9$, $\bar{x} = 39.2$)	84.4	71.5	58.6	45.8	33.4	21.4	10.9	3.3	0.3
S_2 ($e_0 = 72.9$, $\bar{x} = 37.3$)	83.6	69.9	56.4	43.0	30.1	17.9	7.8	1.6	0.1
S_3 ($e_0 = 65.0$, $\bar{x} = 33.4$)	81.6	66.4	51.3	36.6	22.7	10.7	2.7	0.2	0.0
Alternative migration rates: E is local									
$\beta = 0.5\%$, $\gamma = 0.0\%$	82.8	69.2	56.3	44.2	32.8	22.2	12.9	5.4	1.1
$\beta = 1.0\%$, $\gamma = 0.0\%$	79.8	64.7	51.2	39.0	28.1	18.5	10.4	4.2	0.8
$\beta = 0.5\%$, $\gamma = 0.5\%$	85.5	69.9	55.6	42.7	31.0	20.6	11.7	4.8	0.9
$\beta = 1.0\%$, $\gamma = 0.5\%$	82.8	65.8	51.0	38.0	26.8	17.3	9.5	3.8	0.7
Alternative migration rates: E is universal									
$\beta = 0.5\%$, $\gamma = 0.0\%$	82.8	72.7	62.2	51.3	40.0	28.5	17.4	7.7	1.6
$\beta = 1.0\%$, $\gamma = 0.0\%$	79.8	71.5	62.4	52.5	41.8	30.4	18.8	8.5	1.8
$\beta = 0.5\%$, $\gamma = 0.5\%$	85.5	73.5	61.5	49.6	37.8	26.4	15.8	6.8	1.4
$\beta = 1.0\%$, $\gamma = 0.5\%$	82.8	72.7	62.2	51.3	40.0	28.5	17.4	7.7	1.6
Combination that maximizes r									
$S=S_0$, $g = -1.0\%$, $\beta = 0.0\%$, $\gamma = 0.0\%$, E is local	90.3	81.3	71.4	60.5	48.6	35.8	22.7	10.5	2.3
Combination that minimizes r									
$S=S_3$, $g = 2.0\%$, $\beta = 1.0\%$, $\gamma = 0.0\%$, E is local	64.5	43.3	27.7	16.4	8.4	3.2	0.7	0.0	0.0

Note: Standard parameter values are $a = 12$, $g = 0.0\%$, $S = S_0$, $\beta = 0.0\%$, $\gamma = 0.0\%$. These values hold in all cases except as otherwise specified.

The calculation of maximum and minimum r assumes $a = 12$ while all other parameters are allowed to vary.

Table 3: Mean Age of the Remembering Population ($\bar{y}_t$) k Years After an Event E

Parameter specification	k = 0	k = 10	k = 20	k = 30	k = 40	k = 50	k = 60	k = 70	k = 80
Standard values ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
Youngest remembering age									
a = 8 ($\bar{y} = 42.1$)	46.2	51.4	56.7	62.0	67.4	73.0	78.8	85.1	92.0
a = 12 ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
a = 16 ($\bar{y} = 42.1$)	50.4	55.6	60.9	66.3	71.8	77.6	83.8	90.5	98.3
a = 20 ($\bar{y} = 42.1$)	52.5	57.7	63.0	68.5	74.1	80.0	86.4	93.5	101.7
Alternative natural growth rates									
g = - 1.0 % ($\bar{y} = 48.7$)	53.3	57.4	61.7	66.2	71.0	76.1	81.7	88.0	95.1
g = 0.0 % ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
g = 1.0 % ($\bar{y} = 35.8$)	43.5	49.8	56.0	62.2	68.3	74.4	80.8	87.5	95.0
g = 2.0 % ($\bar{y} = 30.2$)	39.2	46.4	53.5	60.3	67.0	73.7	80.4	87.3	94.9
Alternative survival rates									
S ₀ ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
S ₁ ($\bar{y} = 39.2$)	45.4	50.7	56.0	61.4	67.0	72.8	79.1	86.0	94.0
S ₂ ($\bar{y} = 37.3$)	43.5	48.8	54.1	59.6	65.3	71.2	77.7	85.0	93.6
S ₃ ($\bar{y} = 33.4$)	39.7	45.0	50.5	56.1	62.0	68.4	75.5	83.8	92.8
Alternative migration rates: E is local									
$\beta = 0.5\%$, $\gamma = 0.0\%$ ($\bar{y} = 38.9$)	45.9	51.6	57.4	63.1	68.9	74.8	81.0	87.6	95.0
$\beta = 1.0\%$, $\gamma = 0.0\%$ ($\bar{y} = 35.8$)	43.5	49.8	56.0	62.2	68.3	74.4	80.8	87.5	95.0
$\beta = 0.5\%$, $\gamma = 0.5\%$ ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
$\beta = 1.0\%$, $\gamma = 0.5\%$ ($\bar{y} = 38.9$)	45.9	51.6	57.4	63.1	68.9	74.8	81.0	87.6	95.0
Alternative migration rates: E is universal									
$\beta = 0.5\%$, $\gamma = 0.0\%$ ($\bar{y} = 38.9$)	45.9	51.6	57.4	63.1	68.9	74.8	81.0	87.6	95.0
$\beta = 1.0\%$, $\gamma = 0.0\%$ ($\bar{y} = 35.8$)	43.5	49.8	56.0	62.2	68.3	74.4	80.8	87.5	95.0
$\beta = 0.5\%$, $\gamma = 0.5\%$ ($\bar{y} = 42.1$)	48.3	53.5	58.8	64.1	69.6	75.3	81.2	87.7	95.0
$\beta = 1.0\%$, $\gamma = 0.5\%$ ($\bar{y} = 38.9$)	45.9	51.6	57.4	63.1	68.9	74.8	81.0	87.6	95.0
Combination that maximizes r									
S=S ₀ , g= -1.0%, $\beta = 0.0\%$, $\gamma = 0.0\%$, E is local ($\bar{y} = 48.7$)	53.3	57.4	61.7	66.2	71.0	76.1	81.7	88.0	95.1
Combination that minimizes r									
S=S ₃ , g= 2.0%, $\beta = 1.0\%$, $\gamma = 0.0\%$, E is local ($\bar{y} = 22.2$)	31.6	39.2	46.5	53.6	60.6	67.7	75.2	83.6	92.8

Note: See note to Table 2. $\bar{y}$ is mean age of the total population.