Awkward Moments in Teaching Public Finance

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ABSTRACT. While there are many successes for students and teachers to enjoy in modern public finance there remain some ‘awkward moments’ — moments when, you, the student or the teacher, would prefer to be somewhere other than the classroom. This essay discusses three of these ‘moments’ and suggests ways to diminish their awkwardness.

1. Introduction

This lecture is about teaching public finance. There are many moments in the classroom when I feel confident that modern public finance has contributed to a better understanding of events that matter to many people. To take one example, most public finance courses discuss the effects of taxes on labour supply. Teachers often employ a consumption-leisure diagram to show that the deadweight loss of a proportional earnings tax is not zero even if the individual’s (ordinary) labour supply curve is vertical, that is, completely inelastic with respect to the (real) wage rate. If we re-interpret the axes of this diagram to be “years of retirement” rather than “leisure” and “average annual consumption” over the remaining years in the life cycle in place of “consumption”, the same diagram can be used to understand aspects of an individual’s retirement decision (see Burbidge and Robb (1980)). To be more concrete, consider someone aged 50 who expects to live another thirty years. The line $ABC$ in Figure 1 is a possible budget constraint. The kink at point $B$, with 15 years of retirement or retirement at age 65, reflects public and private pension arrangements in Canada and elsewhere. For many people the reward to working another year drops sharply at age 65. When governments in Canada changed the Quebec and Canada Pension Plans in the 1980s to permit individuals to begin Q/CPP retirement pensions as early as age 60 they in effect changed the budget constraint from $ABC$ to $ABDC$. As a consequence, and as the model would predict, there is now a growing spike in the retirement hazard rate at age 60 (see Baker and Benjamin (1999a,b) and Baker, Gruber and Milligan (2000)). That so simple a model can shed light on the socially-important modern trend to retire earlier is gratifying to both students and teachers of public finance.

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While there are many other instances where the fit between the model and the data is tight, or the theory offers sensible insights into individual and market responses to changes in tax rates, I'm sure you would much rather hear about awkward moments — times when you, the teacher or the student, would very much prefer to be somewhere other than the classroom. I begin with the literature on the short-run shifting of the corporate income tax. Next, I deal with some awkward moments in teaching optimal taxation, and then turn to the classic public finance distinction between “efficiency” and “equity”. I should emphasize at the outset that these “awkward moments” are ones that bother me. They may bother others but I don’t know that they do. It wouldn’t surprise me to learn I struggle to understand things that are transparent to everyone else!

2. The Short-Run Shifting of the Corporate Income Tax

Ask any non-economist whether or not firms pay the corporation income tax and without hesitation the answer comes back — “no, we do,” that is, “we” as consumers or “we” as workers. Almost everyone believes that corporations shift any taxes they owe forward onto consumers in higher prices or backward onto their workers in lower wage settlements, or some of both. Many first-year texts use this topic as an illustration of the importance of studying economics. The instructor leads the student through a model of a profit-maximizing firm and then asks what the firm’s best response would be to a proportional tax on its profits. The student’s first reaction is akin to the beliefs of noneconomists; the firm should raise its price. The teacher then points out that by raising its price the firm will only lower its pre-tax profits and would end up with lower after-tax profits for two reasons — pre-tax profits are lower and the government is taking a share of pre-tax profits. The instructor goes on to show that, in fact, the firm’s best response is to leave output and price at their initial levels and simply hand over the tax revenue to the government. More formally, absent the tax, the firm’s problem is to pick output to maximize profits.

\[
\text{Profits} \equiv R(X) - C(X),
\]

where \( R \) denotes total revenue, \( C \) total cost and \( X \) output. If \( X^* \) maximizes profits before tax it will also maximize \((1-t^n)\)Profits, that is, profits after tax, so long as the tax rate is less than unity. From the firm’s point of view the tax rate is a parameter beyond its control.

This result — that, in the short run with a fixed stock of physical capital for each firm, a higher tax on profits must come out of profits — holds not only in the partial-equilibrium model of a profit-maximizing firm but also in general-equilibrium models such as Harberger’s (1962) two-sector model. Most public finance economists today would still agree that the incidence of a profits tax must fall on profits in the short run.

Krzyzaniak and Musgrave (KM, 1963) estimated the following equation on annual data for U.S. manufacturing firms over the period 1935-1942 and 1948-1959:
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\[ r_t = \alpha_0 + \alpha_1 X_t + \alpha_2 X_{t-1} + \alpha_3 \Delta C_{t-1} + \alpha_4 V_{t-1} + \alpha_5 J_t + \alpha_6 G_t + \epsilon_t, \]

where \( r \) is the pre-tax rate of return in manufacturing, \( X \) is the tax rate, \( \Delta C \) is the change in the ratio of consumption to GNP, \( V \) is the ratio of inventories to sales in manufacturing, \( J \) is the ratio of the revenue from all taxes except the corporate income tax to GNP, \( G \) is federal government expenditures and \( \epsilon \) is an error term. Their estimates of \( \alpha_1 \) were such large positive numbers that they concluded that, in the short run, a higher tax on corporate profits raises the pre-tax rate of return by enough that the after-tax rate of return \( \text{rises} \). They interpreted their results as evidence that firms do not maximize profits, at least in the short run. The ensuing debate between KM (1968, 1970) and Cragg, Harberger and Mieszkowski (CHM, 1967, 1970), Gordon (1967, 1968) and others revolved around the inclusion or exclusion of variables designed to control for the state of the business cycle.

This was and still is (see Boadway and Kitchen (1999), pp. 219-220) an awkward moment for teachers of public finance. Almost all of public finance rests on the assumption of full-employment equilibrium. This is implicit in partial-equilibrium models where demand conditions facing the firm are assumed fixed, regardless of what is done with the tax revenue, and explicit in general-equilibrium models like Harberger’s. But the years in KM’s sample were not years of full-employment equilibrium. When the U.S. government raised the corporate income tax, and other taxes, to pay for part of the massive buildup in military spending in the late 1930s and early 1940s the U.S. economy was not at full employment. Consider the saving-investment equilibrium condition in the simplest, closed-economy, macro model we teach our undergraduates.

\[ S = I + G - T, \]

where \( S \) is aggregate saving, \( I \) is investment, \( G \) is government spending and \( T \) is total tax revenue. Hold \( I \) constant for the moment. A large increase in \( G \) that is partly financed by increases in the corporate and other tax rates, increases the budget deficit. This, in turn, stimulates aggregate demand and incomes rise by enough so that the increase in \( S \) on the left-hand side of the equation matches the increase in \( G - T \) on the right-hand side. Both individuals, through their after-tax incomes, and corporations, through their retained earnings and capital consumption allowances, save more. And it is perfectly possible for firms to be short-run profit-maximizers and yet for the increase in corporate tax rates to induce an increase in after-tax real profits — the “burden” of the tax is met out of higher output. The expansion of demand might encourage firms to raise investment expenditures which would strengthen the positive effect of this policy on before-tax and after-tax profits. Indeed, during World War II many governments had to impose controls on private investment to ensure there was enough productive capacity available for the production of war materials.
It is important to realize that there is no such thing as the incidence of the corporate tax or any other broad-based tax. When a broad-based tax is changed something else must change in the government’s budget constraint and the effects of the particular tax change may depend on what the balancing item(s) is(are) in the government’s budget constraint. For example, in the post-war era corporate taxes were often used as an instrument of fiscal policy. If a government raises the corporate tax to reduce the deficit and to “fight inflation”, aggregate demand will decline and thus after-tax corporate profits will fall not only because the corporate tax rate is higher but also because aggregate demand and thus pre-tax corporate profits are lower. To repeat, there is no such thing as the incidence of the corporate (or any other broad-based) tax.¹

A second point comes from inspection of the data on real wages in U.S. manufacturing in the late 1930s and early 1940s. One observes pairs of years with increases in real wages occurring in step with increases in corporate tax rates and after-tax profits. It is unlikely that corporate tax “shifting” occurred in the way KM and most others imagined — that is, by prices rising relative to wage rates.

3. Optimal Taxation

Governments use resources. Taxes are one way to transfer resources from private to public use. One objective of any tax structure is to minimize the welfare loss for given tax revenue. Optimal taxation is the segment of public finance that attempts to determine the relative efficiency of different tax systems.

3.1. Commodity taxation. Most teachers of second-year microeconomics touch on optimal taxation in a partial-equilibrium, one-period model of a price-taking consumer. Let the consumer’s budget constraint be

\[ p_1 x_1 + p_2 x_2 = w, \]

where \( p_i \) are prices, \( x_i \) are consumption levels for two goods, 1 and 2, and \( m \) is the consumer’s money income. If the government raises its required revenue by taxing only good 1 the consumer’s budget constraint becomes

\[ (1 + t_1^c) p_1 x_1 + p_2 x_2 = m, \]

where \( t_1^c \) is the proportional tax rate on good 1. This tax is illustrated in Figure 2. \( AB \) is the no-tax budget constraint. With a commodity tax on good 1 alone the budget constraint becomes \( AC \) and point \( D \) is the consumer’s best choice. The teacher then goes on to show that the consumer would be better off if the government raised its required revenue by taxing both goods at a uniform (lower) rate. Here the budget constraint is:

\[ (1 + t_1^c) p_1 x_1 + (1 + t_2^c) p_2 x_2 = m, \]

with \( t_1^c \) set equal to \( t_2^c \). In Figure 2 budget line \( EF \) is drawn through point \( D \) and parallel to \( AB \). Anywhere along \( EF \) the government collects the same revenue as it does with the tax on commodity 1 alone. Given usual preferences the consumer will be better off at a point like \( G \) with the uniform tax. The point is simply that unless the proportional tax rates are the same, the tax system twists the consumer’s budget constraint and, for given tax revenue, the consumer could be made better off by equalizing the tax rates. In this model a uniform consumption tax is equivalent to a proportional income tax, \( t^m \), because with \( t_1^c = t_2^c \equiv t^c \),

\[
(1 + t^c)(p_1 x_1 + p_2 x_2) = m \quad \text{is the same as} \quad p_1 x_1 + p_2 x_2 = (1 - t^m)m,
\]

so long as we pick \( 1 - t^m = 1/(1 + t^c) \). Since income is an endowment in this model a uniform consumption tax is in effect a lump-sum tax and is therefore “first-best” efficient.

As Corlett and Hague (1953) observed long ago adding a labour-leisure choice to this model makes the optimal tax problem more difficult. Denote the time available to the consumer in this period as \( T \), the time spent at leisure as \( l \) and the wage rate as \( w \), the consumer’s pre-tax budget constraint can be written as

\[
p_1 x_1 + p_2 x_2 + wl = wT.
\]

A proportional tax on \( x_1, x_2 \) and \( l \) would be first-best efficient here just as it was above but the problem is that leisure cannot be taxed directly. We assume that the government can tax commodities \((x_i)\) and earnings \((w(T-l))\), but not leisure. We are now into the world of “second-best” efficiency. Corlett and Hague argued (correctly) that if leisure could not be taxed directly then the optimal commodity tax system would be one in which the good that was more complementary with leisure (tennis racquets) should be taxed at a higher rate than the other good (food). What is meant by “more complementary”?

Atkinson and Stiglitz (1976, 1980) have been widely interpreted as arguing that if leisure is weakly separable from goods then a uniform commodity tax would be (second-best) efficient. What is true in general must be true in particular. Consider the utility function

\[
u(x_1, x_2, l) = x_1^{1/2} x_2^{1/2} + x_1^{1/2} + l^{1/2} \equiv f(x_1, x_2) + g(l).
\]

To say that leisure is weakly separable from goods means that \( u(x_1, x_2, l) \) can be written as \( v(h(x_1, x_2), l) \) so leisure is certainly weakly separable from goods in the example (in fact, it is “strongly” or “additively” separable; see Deaton and Muellbauer (1980, pp. 122, 137)). Almost all teachers of public economics would say that, with this utility function, uniform commodity taxation would be (second-best)
efficient. And yet it is easy to construct numerical examples with this utility function in which the second-best efficient proportional tax rates are unequal. In fact Auerbach (1979) argues that the second-best efficient tax rates can never be equal; it’s efficient always to tax commodity 2 at a higher rate than commodity 1. This is another awkward moment — pretty straight-forward question, and “we” get the wrong answer. Besley and Jewitt (1995) identify a necessary and sufficient condition for the efficiency of uniform commodity taxation. Their condition is equivalent to $(\partial d(u_0, x_1, x_2, l)/\partial x_1)/(\partial d(u_0, x_1, x_2, l)/\partial x_2)$ being independent of leisure, $l$, where the distance function is implicitly defined by\(^2\)

$$u(x_1/d, x_2/d, l/d) = u_0.$$  

The distance function is simply the amount by which the arguments in the utility have to be scaled to yield some particular level of utility $u_0$. In effect, the distance function is a rewriting of the utility function in which we can focus on substitution effects, that is, we can examine the relationship between say goods and leisure holding the utility level constant. It is well known that the deadweight loss of taxes depends solely on substitution effects. In the setting of the optimal commodity tax problem outlined above one has to look at the relationship between $(x_1, x_2)$ and $l$, holding utility constant. As one would expect from the Corlett-Hague intuition, if goods are equally complementary with leisure, there’s no point taxing them at different rates and uniform commodity taxation is efficient.

3.2. Interest taxation. In a partial-equilibrium, pure life-cycle model (with no bequests or inheritances) it is also well known that a proportional earnings tax is equivalent to a proportional consumption tax. And it is very common for public economics teachers to use a two-period consumption model (with fixed real earnings levels in each period) — see Figure 3 — to show that both consumption and wage taxation are more efficient than a proportional interest tax. For given tax revenue, the interest tax twists the budget constraint so that it is possible to make the consumer better off with either a consumption tax or a wage tax. Simple though the Figure 3 diagram is, it has had a huge impact on the way policy makers think about the relative efficiencies of consumption and income taxes (note both wage and interest income are included in the income tax base). Much of the impetus to move away from income taxation towards consumption taxation derives from the image of the interest tax twisting the consumer’s budget constraint in a way that wage or consumption taxes do not, although one should not underplay the influence of general-equilibrium modelers (e.g. Summers (1981) and Auerbach, Kotlikoff and Skinner (1983)).

I am in the habit of working through numerical examples with my public finance classes. A few years ago we were gathered around the computer and I was illustrating optimal tax configurations in a two-period life-cycle model exactly like the diagram

\(^2\)It is easy to verify that this ratio is not independent of $l$ in the example above; see Alvarez et al. (1992).
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in Figure 3 except that there was a labour-leisure choice in each period. I was so persuaded that interest taxes were inefficient that I said something like let’s remove the restriction that interest taxes are zero and we’ll see that the optimal interest tax rate is indeed zero. The computer program then proceeded to generate higher utility levels for a given present value of tax revenue by making the interest tax rate positive. This was an awkward moment! At the time I was convinced there was an error in the computer program. The answer the class and I worked out turned out to be more complicated. Briefly, this is the story.3

Many people think that in the context of a pure life-cycle model the conditions under which it is efficient to tax goods at a uniform rate are the same as the conditions under which leisure should be taxed at a uniform rate, and they are correct. The problem, as I have noted, is that leisure cannot be taxed — this is what makes optimal tax problems non-trivial. Earnings can be taxed but earnings are the product of wage rates and time spent at work, that is, the time endowment less time at leisure. As far as I know, and unlike consumption taxation, there are no general results for the conditions under which uniform earnings taxation is efficient. All we have are special cases. We know that if we assume preferences are additively separable over time, the within-period utility function for consumption and leisure is the same for every period, the real wage rate is the same for every period and the person’s utility discount rate is the same as the real interest rate then the person’s optimal leisure (and consumption) plan is flat over the life cycle. Given the perfect symmetry between work in periods 1 and 2 in this setting surely it’s efficient to tax the earnings in the two periods equally. Indeed, this is correct. And if we break the symmetry by making the real interest rate larger than the person’s discount rate then the optimal leisure (and consumption) plans slope upwards with age. The individual wishes to work harder earlier in the life cycle and it is efficient to have wage tax rates fall over the life cycle.

How can it be efficient to have a positive interest tax rate? Consider the budget constraint of a two-period consumer facing age-conditioned proportional consumption and wage tax rates and an interest tax rate.

\[
(1 + t^c_1)c_1 + \frac{(1 + t^w_1)c_2}{1 + r(1 - t^r)} = (1 - t^w_1)w(1 - l_1) + \frac{(1 - t^w_2)w(1 - l_2)}{1 + r(1 - t^r)}
\]

With a full set of consumption and wage tax rates there can be no role for an interest subsidy or tax because we already have an instrument to change the price of consumption and/or earnings in each period — the interest tax rate is redundant. But if the government is constrained not to age condition wage tax rates, that is, the government is forced to set \( t^w_1 = t^w_2 \) then in circumstances when it would be efficient to make \( t^w_1 > t^w_2 \), or \( (1 - t^w_1) < (1 - t^w_2) \), inspection of the right-hand side of the budget constraint reveals that \( t^r > 0 \) (imperfectly) mimics this outcome.4

All of this wouldn’t matter much except that Summers’s famous AER article used a life-cycle growth model to attack interest taxation. Among other things Summers

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3 For the longer version see Alvarez et al. (1992).

4 I say “imperfectly” because \( t^r \) does distort the consumer’s consumption plan.
showed that a wage tax regime would generate higher steady-state utility than an income tax regime. Summers abstracted from a labour-leisure choice. In the well-known diagram in AKS, whose model is much like Summers’s except that AKS do admit a labour-leisure choice, the income tax steady state has *higher* utility than the wage tax steady state. It is difficult to be sure exactly why results on the relative efficiencies of wage and income taxation differ between these two landmark articles but I am reasonably confident that, on AKS’s assumptions, there is a role for an interest tax in AKS’s model.

4. Equity and Efficiency

The most awkward moment in my taxation classes occurs at the point when I switch from talking about the relative efficiencies of various tax systems to discussing horizontal and vertical equity and the fairness of various tax systems. Most years I’ll get the students to go over to the bookstore to pick up the *Guide* for the personal income tax. The 1999 edition had 40 pages — very little of it was about economic efficiency; most of it was about equity. Several pages were devoted to defining the income tax base and what constitutes taxable income. Married couples are treated differently from those who are unmarried, families with children are treated differently from families without children, older people are treated differently from younger people, higher income households typically face higher marginal tax rates than lower income households, and so on. The Guide is an attempt to convey the details of the Income Tax Act which in turn is based on decades of experimentation with income taxation in Canada and elsewhere. While economic models of efficient tax structures assume agents do not care a whit about anyone else it is obvious that throughout history people have looked sideways — they have cared not only about what they “get” but also about what others they know (inside or outside of their families) “get”.

Consider two alternative actions that might be taken by the dean of an imaginary social science faculty composed of equal parts sociologists, political scientists and economists. To balance his budget the dean must cut salaries by 2 percent. One alternative, call it *A*, is to cut everyone’s salary by 2 percent. Another alternative, *B*, which let’s suppose has the identical budgetary impact, is to cut the salaries of the political scientists by 4 percent, economists’ salaries by 2 percent and to leave the sociologists’ salaries unchanged. If economists had preferences like those of the agents that populate their models they would be indifferent between *A* and *B*. I have witnessed *A* — there’s some grumbling but life goes on. I’m quite sure that if *B* were implemented some of the economists would be smiling, inwardly or outwardly, about their treatment relative to that of the political scientists and every economist would be screaming about the inequitable treatment of economists relative to sociologists.

Much of (public) economics deals with the contradiction between the efficiency model and the (inferred) equity model by means of social welfare functions. The story is that economic women and men are totally selfish but their governors have preferences for redistribution from the better off to those who are worse off. All economic models must abstract from reality to be useful. Is a social welfare function
a convenient analytical device or a misleading illusion?

Consider the following very simple two-person example. Person 1 has higher ability and thus works at a higher wage rate than person 2. Each person’s utility function is

\[ U_i = C_i + 2L_i^{1/2}, \quad C_i \geq 0, \quad 0 \leq L_i \leq 1, \]

where \( C \) is consumption and \( L \) is leisure. Person i’s budget constraint is

\[ C_i = w_i(1 - L_i) - T_i, \]

where \( w_i \) is the wage rate and \( T_i \) is a lump-sum tax levied by the government. Assuming we are not at the corner solution for quasilinear preferences, \( L_i = 1/w_i^2 \) (so \( w_i \) must be chosen to be greater than unity) and each person’s indirect utility function is

\[ V_i(w_i, T_i) = \frac{w_i^2 + 1}{w_i} - T_i. \]

If the government’s budget constraint is \( T_1 + T_2 = 0 \) the equation of the utility possibility frontier is

\[ V_1 + V_2 = \frac{w_1^2 + 1}{w_1} + \frac{w_2^2 + 1}{w_2}. \]

In the standard model the government chooses taxes to maximize a social welfare function such as the Rawlsian one, \( \min(V_1, V_2) \), subject to the constraint set by the utility possibility frontier. This yields

\[ T_i = \frac{w_i^2 + 1}{2w_i} - \frac{w_j^2 + 1}{2w_j}, \quad i \neq j. \]

Since \( w_1 > w_2 \geq 1 \), the optimal tax configuration has \( T_1 > 0 \) and \( T_2 < 0 \), that is, redistribution occurs from the higher ability person to the lower ability person. Note that this action by the “equity”-seeking government makes the higher ability person worse off.

Contrast this outcome with a setting in which the agents themselves are more or less averse to inequality. Suppose one person cannot observe the other person’s utility level or work effort but that each can observe the other’s consumption. Let the utility functions assume the particular form

\[ U_i = C_i + 2L_i^{1/2} - \alpha|C_i - C_j|, \quad i \neq j, \quad \alpha > 0. \]

In the appendix I establish the results in Table 1.
Table 1: Summary of Optimal Choices for Persons 1 and 2

<table>
<thead>
<tr>
<th>Range</th>
<th>$T^*_1$</th>
<th>$L^*_1$</th>
<th>$L^*_2$</th>
<th>$C^*_1$</th>
<th>$C^*_2$</th>
<th>$U_1$</th>
<th>$U_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha &gt; 1/2$</td>
<td>$\frac{1}{2} \left( \frac{w_1 (1 - L^<em>_1)}{w_2 (1 - L^</em>_2)} \right)$</td>
<td>$\frac{4}{w_1}$</td>
<td>$\frac{4}{w_2}$</td>
<td>$C^<em>_1 = C^</em>_2$</td>
<td>$U_1 &lt; U_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/2 \geq \alpha \geq \alpha^*$</td>
<td>0</td>
<td>$\frac{1}{(1-\alpha^2)w_1^2}$</td>
<td>$\frac{1}{(1+\alpha^2)w_2^2}$</td>
<td>$C^<em>_1 = C^</em>_2$</td>
<td>$U_1 &gt; U_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/2 \geq \alpha$ and $\alpha^* &gt; \alpha$</td>
<td>0</td>
<td>$\frac{1}{(1-\alpha)w_1}$</td>
<td>$\frac{1}{(1+\alpha)w_2}$</td>
<td>$C^<em>_1 &gt; C^</em>_2$</td>
<td>$U_1 &gt; U_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. $T^*_1$ is the optimal transfer from person 1 to person 2.
2. $\alpha^*$ solves equation (5) in the appendix.

Even though a Rawlsian social welfare function embodies an incredibly strong urge to equalize this example shows endowing the agents directly with a necessarily weaker urge to equalize may have more dramatic results. At $\alpha = 0$ we have the standard result that the higher ability person works harder, and enjoys higher consumption and utility. As $\alpha$ is increased there is a range where the high-ability person works less hard, the low-ability person works harder and the relative ranking of consumption and utility are like those with $\alpha = 0$. If $\alpha^* < 1/2$, as $\alpha$ rises above 0, there will come a point where $1/2 \geq \alpha \geq \alpha^*$. In this range earnings and consumption are equalized but the high-ability person still enjoys higher utility. When $\alpha > 1/2$ private charity is optimal — person 1 finds it optimal to transfer wealth to person until their consumption levels are equalized. In this setting the high-ability person works harder than the low-ability person and thus has lower utility. Admitting the possibility of interdependent preferences may help to explain why some higher income individuals support private charities and public programs like progressive taxation that redistribute from higher income to lower income households. This contrasts rather dramatically with the standard redistribution story, described above, in which redistribution by the government to maximize its social welfare function hurts the high-ability person.

While this example is suggestive it would be a major error to put too much weight on it. There is a long and well-grounded tradition in economics about not changing preferences every time we have an awkward fact to explain. The inter-dependent preference solution to the equity-efficiency chasm will be useful only if researchers can find a representation of preferences that is consistent with many facts about human societies. It is clear that as in so many other areas of economics taking individual heterogeneity into account will be extremely important.

5. Conclusions

I wish I had written Musgrave’s *Theory of Public Finance* (1959). But expositional clarity was purchased at a price. The tidy subdivisions of allocation, distribution and stabilization have occasionally cut us off from a clearer understanding of how the world works and what embodies sensible public policy. This morning Peter Howitt
(2000) argued persuasively for breaking down the borders between growth theory and the theory of economic development. This lecture strikes the same chord. It would be much more fun to teach public finance theories and statistical analyses that integrate micro and macro, and efficiency and equity.
Person 1 is the high ability (high wage) person. She chooses consumption, $C_1$, and leisure, $L_1$, to maximize her utility. In some circumstances she may also want to make a positive transfer, $T_1$, to person 2. Thus I can write person 1’s budget constraint as $C_1 = w_1(1 - L_1) - T_1$ and person 2’s budget constraint as $C_2 = w_2(1 - L_2) + T_1$. Assuming person 1 acts to maximize her utility taking as given the choices of person 2, I can write person 1’s optimization problem as follows.

$$\max_{L_1, T_1} \mathcal{L} \equiv w_1(1 - L_1) - T_1 + 2L_1^{1/2} - \alpha |w_1(1 - L_1) - w_2(1 - L_2) - 2T_1|$$

The argument of the absolute value function is $C_1 - C_2$. Denote the optimal consumption values by $C^*_i$. Since the derivative of the absolute value function is undefined at zero, but its left and right derivatives exist, and in some circumstances it happens that $C^*_1 = C^*_2$ is optimal, I use left and right derivatives to write the Kuhn-Tucker conditions. Assuming $C^*_1 \geq C^*_2$, they are:

$$T_1(2\alpha - 1) = 0, \text{ with } (2\alpha - 1) \leq 0, \text{ for } C^*_1 > C^*_2$$

$$(\frac{\partial \mathcal{L}}{\partial T_1})^R \leq 0, \text{ so } T_1(-1 - 2\alpha) \leq 0, \text{ for } C^*_1 = C^*_2$$

$$(\frac{\partial \mathcal{L}}{\partial T_1})^L \geq 0, \text{ so } T_1(-1 + 2\alpha) \geq 0, \text{ for } C^*_1 = C^*_2$$

and

$$-(1 - \alpha)w_1 + L_1^{-1/2} = 0, \text{ for } C^*_1 > C^*_2$$

$$(\frac{\partial \mathcal{L}}{\partial L_1})^R \leq 0, \text{ so } -(1 + \alpha)w_1 + L_1^{-1/2} \leq 0, \text{ for } C^*_1 = C^*_2$$

$$(\frac{\partial \mathcal{L}}{\partial L_1})^L \geq 0, \text{ so } -(1 - \alpha)w_1 + L_1^{-1/2} \geq 0, \text{ for } C^*_1 = C^*_2.$$  

In writing (2) I have used the fact that with this utility function $L_1 = 0$ can never be optimal. From (2) one can see that

$$L_1 \left\{ \begin{array}{ll} = 1/[(1 - \alpha)w_1]^2, & \text{for } C^*_1 > C^*_2 \\ \leq 1/[(1 - \alpha)w_1]^2, & \text{for } C^*_1 = C^*_2 \\ \geq 1/[(1 + \alpha)w_1]^2, & \text{for } C^*_1 = C^*_2 \end{array} \right. \quad (3)$$

Likewise the optimization problem for person 2 yields

$$L_2 \left\{ \begin{array}{ll} = 1/[(1 + \alpha)w_2]^2, & \text{for } C^*_1 > C^*_2 \\ \leq 1/[(1 - \alpha)w_2]^2, & \text{for } C^*_1 = C^*_2 \\ \geq 1/[(1 + \alpha)w_2]^2, & \text{for } C^*_1 = C^*_2 \end{array} \right. \quad (4)$$

Looking at the first line of (1), $T_1 = 0$ if $0 \leq \alpha < 1/2$ and $C^*_1 > C^*_2$, and if $\alpha > 1/2$, $C^*_1 > C^*_2$ cannot be optimal. Since $w_1 > w_2$ and the two people have the same utility function $T_1$ must be positive if $C^*_1 = C^*_2$. Combining the lines of (1) one can deduce then that if $\alpha > 1/2$ it is optimal to choose $T_1 > 0$ to make $C^*_1 = C^*_2$. Thus $\alpha > 1/2$ is one case we have to consider. When $\alpha$ is small, $T_1 = 0$ and the earnings of person
That from (5) I know that when inequality succeed and she works harder than person 2 because she has a higher wage rate. When \( \alpha \) is increased it may happen that \( C^*_1 = C^*_2 \) for \( 1/2 \geq \alpha \geq \alpha^* \) where \( \alpha^* \) is the value of \( \alpha \) that solves the following equation

\[
w_1 \left(1 - \frac{1}{[(1 - \alpha)w_1]^2}\right) = w_2 \left(1 - \frac{1}{[(1 + \alpha)w_2]^2}\right).
\]

(5)

Here it’s optimal to set \( L_1 = 1/[(1 - \alpha^*)w_1]^2 \), \( L_2 = 1/[(1 + \alpha^*)w_2]^2 \) and \( T_1 = 0 \). The \( \alpha \geq \alpha^* \) inequality follows from (3) and (4). The final case is \( 1/2 \geq \alpha \) and \( \alpha^* > \alpha \). Here the Kuhn-Tucker conditions imply \( L_1 = 1/[(1 - \alpha)w_1]^2 \), \( L_2 = 1/[(1 + \alpha)w_2]^2 \) and \( T_1 = 0 \). At \( \alpha = 1/2 \) person 1 is indifferent between giving money to person 2 or not doing so. I assume she chooses \( T_1 = 0 \).

To complete the demand system we have to work out optimal values for \( L_i \) (call them \( L^*_i \)) when \( \alpha > 1/2 \). In this case, as I showed above, \( T_1 > 0 \) and \( C^*_1 = C^*_2 \). The latter equality implies that

\[
T_1 = \frac{1}{2}(w_1(1 - L^*_1) - w_2(1 - L^*_2)),
\]

that is, the transfer from person 1 to person 2 equalizes after-transfer incomes. So person 1’s optimization problem can be rewritten as

\[
\text{Max}_{L_1} \frac{1}{2} [w_1(1 - L_1) + w_2(1 - L_2)] + 2L_1^{1/2},
\]

from which it follows that \( L^*_1 = 4/w_1^2 \). Similarly it follows that \( L^*_2 = 4/w_2^2 \). The results to this point are summarized in the first five columns of Table 1 in the text.

It remains to prove the results in the last column. That \( U_1 < U_2 \) when \( \alpha > 1/2 \) follows directly from \( C^*_1 = C^*_2 \) and \( L^*_1 < L^*_2 \). Person 1’s efforts to reduce consumption inequality succeed and she works harder than person 2 because she has a higher wage rate. When \( 1/2 \geq \alpha \geq \alpha^* \), \( C^*_1 = C^*_2 \) so \( w_1(1 - L^*_1) = w_2(1 - L^*_2) \). With \( w_1 > w_2 \) it must be that \( L^*_1 > L^*_2 \) and thus \( U_1 > U_2 \). To prove that \( U_1 > U_2 \) when \( 1/2 \geq \alpha \) and \( \alpha^* > \alpha \) is more work.

Define

\[
f(\alpha) \equiv w_1 \left(1 - \frac{1}{[(1 - \alpha)w_1]^2}\right) - w_2 \left(1 - \frac{1}{[(1 + \alpha)w_2]^2}\right).
\]

From (5) I know \( f(\alpha^*) = 0 \). Now note that

\[
U_1 - U_2 \equiv g(\alpha) = f(\alpha) + 2 \left(\frac{1}{(1 - \alpha)w_1} - \frac{1}{(1 + \alpha)w_2}\right) \equiv f(\alpha) + h(\alpha).
\]

That \( w_1 > w_2 \) implies \( f(0) > 0 \). Since \( L_i \leq 1 \) it must be true that \( w_i > 1 \) and thus \( 0 < g(0) < f(0) \). Now
f'(a) = -\frac{2}{w_1(1-a)^3} - \frac{2}{w_2(1+a)^3} < 0

and

h''(a) = \frac{2}{(1-a)^2 w_1} + \frac{2}{(1+a)^2 w_2} > 0,

so that \( g(\alpha) \) must fall more slowly than \( f(\alpha) \), for every value of \( \alpha \). Let \( \alpha^\# \) solve \( h(\alpha) = 0 \). If I can show that \( f(\alpha^\#) > 0 \) then I know that if it ever happens that \( g(\alpha) = 0 \) it is at a value of \( \alpha \) that is greater than \( \alpha^* \). In other words \( U_1 - U_2 > 0 \) when \( \alpha^* > \alpha \). Solving \( h(\alpha) = 0 \) obtain

\[ \alpha^\# = (w_1 - w_2)/(w_1 + w_2). \]

Then

\[
\begin{align*}
  f(\alpha^\#) &= \frac{1}{4} \left( 4w_1^2 w_2^2 - w_1^2 - 2w_1 w_2 - w_2^2 \right) \frac{w_1 - w_2}{w_1^2 w_2^2} \\
  &= \frac{1}{4} \left( w_1^2(w_2^2 - 1) + 2w_1 w_2(w_1 w_2 - 1) + w_2^2(w_1^2 - 1) \right) \frac{w_1 - w_2}{w_1^2 w_2^2} \\
  &> 0, \text{ at } w_1 > w_2 > 1.
\end{align*}
\]

This completes the proofs for the results in Table 1.
References


Figure 1: Consumption and Retirement

- Annual Consumption X $10^{-4}$
- Years of Retirement

Points: A, B, C, D
Figure 3: Interest vs. Consumption Taxation
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