Inventories, Sticky Prices, and the Propagation of Nominal Shocks

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Abstract

Post-war business cycle fluctuations of output and inflation are remarkably persistent. Many recent sticky-price monetary business cycle models, however, grossly underpredict this persistence. We assess whether adding inventories to a standard sticky-price model raises the persistence of output and inflation. For this addition, we consider three different frameworks: a linear-quadratic inventory model, a factor of production model, and a shopping-cost model. We find that adding inventories increases the persistence of output and inflation, but that the increase is smaller for inflation. Overall, the shopping-cost model best explains the persistence of output and inflation.


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1. Introduction

Post-war US business cycle fluctuations of output and inflation are remarkably persistent. A number of recent papers argue that existing monetary business cycle models with explicit microfoundations fail to explain the persistence of output and inflation. For example, Ascari (2000) and Chari, Kehoe, and McGrattan (2000) show that monetary business cycle models with a price staggering version of the Taylor (1980) overlapping contracts fail to explain persistent output fluctuations. Nelson (1998) documents that several existing monetary business cycle models fail to explain persistent inflation changes.

Our objective is to determine whether adding inventories to a standard sticky-price monetary business cycle model raises the persistence of output and inflation.\footnote{Several papers alter the basic monetary business cycle model to explain higher persistence. To enhance the persistence of output, some papers add staggered wage contracts (Andersen 1998, Edge 2002, Erceg 1997, and Huang and Liu 2002), change the demand structure (Bergin and Feenstra 2000), and alter the production structure (Huang and Liu 2001 and Kiley 2000). To enhance the persistence of inflation, some papers add monetary policy rules (Dittmar, Gavin, and Kydland 2001 and Ireland 2001, 2003) and introduce relative real wage concerns (Fuhrer and Moore 1995). Finally, to enhance the persistence of both output and inflation, some papers introduce relative real wage concerns (Ascari and Garcia 1999) and consider habit formation and capacity utilization (Christiano, Eichenbaum, and Evans 2001).} Research on inventories has been surveyed by Blinder and Maccini (1991) and Ramey and West (1999). In this literature, some studies examine the relation between inventories and the business cycle (Bils and Kahn 2000, Blinder and Fischer 1981, Fisher and Hornstein 2000, West 1990), between inventories and sticky prices (Blinder 1982, Borenstein and Shepard 2002, Hornstein and Sarte 1998), and between inventories and costly price changes (Aguirregabiria 1999). We follow these studies and focus on the impact of inventories for several reasons. First, Ramey and West (1999) document that, although changes in inventories form on average less than one percent of gross domestic product, reductions in inventories arithmetically account for about 49 percent of the fall in gross domestic product during post-war US recessions. Second, Blinder and Fischer (1981) argue that the gradual adjustment of the stock of inventories is responsible for lasting real effects of
changes in the stock of money. Finally, Blinder (1982) argues that inventories generate (real) price stickiness.

The last two reasons suggest that the gradual adjustment of inventory stocks is an explanation for the sluggishness of both output and inflation changes. In the terminology of Ball and Romer (1990), inventories create a real rigidity. They write “Researchers have presented a wide range of explanations for wage and price rigidities: examples include implicit contracts, customer markets, social customs, efficiency wages, inventory models, and counter-cyclical markups” (page 183). In other words, the effects of money growth shocks on the real economy created by nominal rigidities become quantitatively important and persistent with inventories.

To achieve our objective, we compare the persistence of output and inflation in monetary business cycle models with and without inventories. We evaluate whether adding inventories raises the persistence by directly comparing the sample autocorrelations of output and inflation produced by the different models. We also evaluate whether these sample autocorrelations replicate the autocorrelations calculated in post-war US data. In addition, we verify whether the models with inventories reproduce three features of the data: sales are less volatile than output and changes in inventories are less volatile than output and procyclical.

Section 2 presents our baseline sticky-price model without inventories. It consists of an artificial economy populated by an infinitely-lived representative consumer, a representative competitive retailer, monopolistically competitive producers, and a monetary authority. The consumer purchases an aggregate good from the retailer. The retailer purchases individual goods from producers and aggregates them. As both the consumer and the retailer are price-takers, our economy is equivalent to one where the consumer purchases individual goods directly from producers. We nevertheless introduce the retailer because this modeling choice simplifies the exposition. Individual goods are produced by monopolistically competitive producers using labor and capital. Although we are mainly concerned with the propagation of nominal shocks, we introduce persistent productivity shocks. These shocks will prove useful in the inventory models. Also, as in Ireland (2001), producers find it costly to adjust nominal prices. We find that the nominal rigidity explains
lasting effects of money growth shocks on output and inflation. The persistence of these effects, however, is much smaller than that found in post-war US data.

Sections 3, 4, and 5 verify whether adding inventories to the baseline model enhances the persistence of output and inflation. In each of these inventory models, only producers hold inventories, while the retailer does not. Ramey and West (1999) document that, for 1995, about 37 percent of inventories were held in manufacturing and 52 percent were held in either retail or wholesale trade. We abstract from inventories in the retail sector for two reasons. First, we introduce the retailer only to simplify the exposition. Second, we are interested in the interaction between inventories and pricing decisions of monopolistic producers. In doing so, we follow Blinder and Fischer (1981) and Hornstein and Sarte (1998).

Section 3 discusses the persistence of output and inflation in a model with inventories that share several features with the linear-quadratic model of West (1990). In this model, producers manage an inventory stock of goods, but face costs of changing the level of production and costs of deviating from a ratio of sales to inventories. The first cost provides a production smoothing motive and the second represents stockout costs. Our main empirical findings are as follows. First, adding inventories raises the persistence of output and inflation, but the effect is too small for inflation. Second, the model counterfactually predicts that sales are more volatile than output and that changes in inventories are countercyclical.

Section 4 discusses the persistence of output and inflation in a factor of production model that embodies a feature found in the model of Kydland and Prescott (1982). In this model, producers manage an inventory stock of goods that is a direct input in production. The inventory stock is a production input because it helps economize on the cost of restocking and the cost of shifting production from one type of good to another. The main empirical findings are similar to those of the linear-quadratic model. That is, adding inventories adequately raises persistence of output and inadequately raises that of inflation. Also, the model counterfactually predicts that sales are more volatile than output and that changes in inventories are countercyclical.

Section 5 discusses the persistence of output and inflation in a shopping-cost model
that shares features with the model of Bils and Kahn (2000). In this model, the consumer finds shopping activities costly. A larger stock of inventory augments the stock of available goods, which makes it easier to shop. The empirical findings of the shopping-cost model differ from those of our previous inventory models. Adding inventories using the shopping-cost model significantly raises the persistence of both output and inflation. Also, the model correctly predicts that changes in inventories are procyclical, but incorrectly predicts that sales are more volatile than output.

2. The Baseline Model

The baseline model does not include inventories. It depicts a stochastic economy populated by an infinitely lived representative consumer, a representative retailer, a continuum of monopolistically competitive producers indexed by $i \in [0, 1]$, and a monetary authority. The retailer aggregates individual goods, and sells the aggregate to the consumer. Production of individual goods requires both labor and capital, and producers find it costly to change nominal prices. The monetary authority supplies money according to a stochastic rule. Finally, the notation follows that of Chari, Kehoe, and McGrattan (2000). That is, in each period the economy experiences an event $h_t$. The history of events at time $t$ is $h_t = (h_0, h_1, \ldots, h_t)$ and $h_0$ is given. The probability at period 0 of history $h^t$ is $\pi(h^t)$ and the conditional probability of history $h^{t+1}$ at period $t$ is $\pi(h^{t+1}|h^t)$, where $\pi(h^{t+1}) = \pi(h^{t+1}|h^t)\pi(h^t)$ and $\pi(h^0) = 1$.

2.1 The Consumer

The representative consumer chooses consumption, hours worked, investment, and asset and money holdings to maximize expected lifetime utility

$$\sum_{t=0}^{\infty} \sum_{h^t} \beta^t \pi(h^t) U \left( C(h^t), M(h^t)/P(h^t), N(h^t) \right)$$

subject to the budget constraint

$$P(h^t) \left[ C(h^t) + I(h^t) \right] + M(h^t) + \sum_{h_{t+1}} q(h_{t+1}|h^t) B(h_{t+1})$$

$$P(h^t) \left[ w(h^t) N(h^t) + r^K(h^t) K(h_{t-1}) \right] + M(h_{t-1}) + B(h^t) + T(h^t) + \Pi(h^t),$$

(2)
where $C$ denotes consumption, $M$ is nominal money balances, $P$ is the aggregate price level, $N$ is hours worked, $I$ is investment, $K$ is the capital stock, $T$ is nominal transfers, $w$ is the real wage rate, $r^k$ is the rental rate of capital, and $\Pi$ is the aggregate of all profits. Also, the consumer purchases contingent one-period nominal bonds $B$, but faces the borrowing constraint $B \geq B$ for some large negative number $B$. The price $q(h^t+1|h^t)$ denotes the price of a bond purchased in period $t$ that pays one dollar in period $t+1$ if state $h_{t+1}$ is realized. The period utility is given by

$$U(C, M/P, N) = \frac{1}{1-\sigma} \left( \left[ \omega C^{\frac{1}{x-1}} + (1-\omega)(M/P)^{\frac{1}{x-1}} \right]^{\frac{x}{x-1}} (1-N)^{\psi} \right)^{1-\sigma}.$$  

The capital stock evolves according to

$$K(h^t) = I(h^t) + (1-\delta)K(h^{t-1}) - \frac{\nu}{2} \left( \frac{I(h^t)}{K(h^{t-1})} - \delta \right)^2 K(h^{t-1}), \quad (3)$$

where the last term of equation (3) denotes capital adjustment costs. As in Chari, Kehoe, and McGrattan (2000) and Ireland (2000), the adjustment cost is used to dampen the extreme volatility of investment produced by some of the models considered.

### 2.2 The Retailer

The competitive retailer chooses purchases to maximize profits

$$P(h^t)G(h^t) - \int p_i(h^t) s_i(h^t) \, di, \quad (4)$$

subject to the aggregation technology

$$G(h^t) = \left[ \int g_i(h^t)^{\frac{\rho-1}{\rho}} \, di \right]^{\frac{\rho}{\rho-1}}, \quad (5)$$

where $G$ denotes the quantity of aggregate goods sold to the consumer, $p_i$ is the sales price for good $i$, and $s_i = g_i$ is the quantity purchased of good $i$.

The retailer’s first-order conditions imply the goods demand function

$$s_i^d(h^t) = \left[ \frac{P(h^t)}{p_i(h^t)} \right]^{\theta} G(h^t). \quad (6)$$
The demand functions for all goods and the retailer’s zero-profit condition yield the aggregate price index
\[ P(h^t) = \left[ \int p_i(h^t)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \] (7)

2.3 Producers

Monopolistic producer \( i \) chooses labor, capital, and prices to maximize expected discounted profits
\[ \sum_{t=0}^{\infty} \sum_{h^t} q(h^t) \left( p_i(h^t)s_i^d(h^t) - P(h^t) \left[ w(h^t)n_i(h^t) + r^k(h^t)k_i(h^t) \right] \right), \] (8)
subject to the production technology
\[ y_i(h^t) = z(h^t)k_i(h^t)^\alpha n_i(h^t)^{1-\alpha}, \] (9)
the definition of net output
\[ y_i^n(h^t) = y_i(h^t) - \frac{\phi_p}{2} \left( \frac{p_i(h^t)}{\bar{\mu}p_i(h^{t-1})} - 1 \right)^2 y_i(h^t), \] (10)
and the demand for good \( i \) depicted in equation (6), where \( n_i \) is labor, \( k_i \) is capital, \( y_i \) is gross output, \( y_i^n \) is net output, \( z \) is an aggregate total factor productivity (TFP) shock, and \( \bar{\mu} \) denotes the steady-state level of inflation. The price \( q(h^t) = q(h^t|h^{t-1})q(h^{t-1}) \) where \( q(h^0) = 1 \) is constructed from the consumer’s first-order conditions. Also, as shown in equation (10), price adjustments are costly and drive a gap between net and gross output.

The price adjustment costs guarantees nominal price rigidity.

Finally, the TFP shock evolves as
\[ \ln \left( z(h^t) \right) = (1 - \rho_z) \ln(\bar{z}) + \rho_z \ln \left( z(h^{t-1}) \right) + \epsilon_{zt}, \] (11)
where \( \bar{z} \) is the mean level of TFP and \( \epsilon_z \) is a mean zero random variable with variance \( \sigma_z^2 \).
2.4 The Monetary Authority

The monetary authority provides nominal transfers according to

\[ T(h^t) = M(h^t) - M(h^{t-1}). \]  

(12)

The growth rate of money, \( \mu(h^t) = \ln \left( \frac{M(h^t)}{M(h^{t-1})} \right) \), evolves as

\[ \mu(h^t) = (1 - \rho_\mu) \ln(\bar{\mu}) + \rho_\mu \mu(h^{t-1}) + \epsilon_\mu, \]  

(13)

where \( \epsilon_\mu \) is a mean zero random variable with variance \( \sigma_\mu^2 \).

2.5 Market Clearing and Aggregation

Clearing of the bond, capital, and labor markets requires

\[ B(h^t) = 0, \]  

(14.1)

\[ K(h^{t-1}) = \int k_i(h^t) \, di. \]  

(14.2)

\[ N(h^t) = \int n_i(h^t) \, di. \]  

(14.3)

Note that, as individual producers face identical problems, they will charge identical prices. Our symmetric equilibrium thus imposes that \( P(h^t) = p_i(h^t) \) and \( s_i(h^t) = g_i(h^t) \). This implies that \( K(h^{t-1}) = k_i(h^t) \), \( N(h^t) = n_i(h^t) \), \( Y(h^t) = \int y_i(h^t) \, di = y_i(h^t) \), \( Y^n(h^t) = \int y^n_i(h^t) \, di = y^n_i(h^t) \), and \( S(h^t) = \int s_i(h^t) \, di = s_i(h^t) \). Then, the goods market clearing conditions simplify to

\[ C(h^t) + I(h^t) = G(h^t). \]  

(14.4)

\[ Y^n(h^t) = Y(h^t) - \frac{\phi_P}{2} \left( \frac{P(h^t)}{\mu P(h^{t-1})} - 1 \right)^2 Y(h^t), \]  

(14.5)

\[ G(h^t) = S(h^t), \]  

(14.6)

\[ S(h^t) = Y^n(h^t). \]  

(14.7)
2.6 Simulation Method and Benchmark Parameter Values

The baseline model does not have an analytical solution for general values of the underlying parameters. Instead, we find an approximate solution using the method described in King, Plosser, and Rebelo (2002). This method requires that values be assigned to all parameters.

Table 1 displays parameter values for the different models. For the baseline model, we set several parameters to the values used in Chari, Kehoe, and McGrattan (2000): \( \chi = 0.39, \omega = 0.94, \delta = 0.025, \alpha = 0.36, \theta = 10, \bar{z} = 1, \) and \( \bar{\mu} = 1. \) Also, we follow their guidelines and set \( \psi = 1.7119 \) to ensure that hours worked are 30 percent of the time endowment. The source of nominal rigidity in our baseline model differs from that used in Chari, Kehoe, and McGrattan (2000). This influences the values of both \( \beta \) and \( \phi_p. \)

We follow Kydland and Prescott (1982) and set \( \beta = 0.99. \) We also follow Ireland (2001) and use his estimated value of \( \phi_p \) for the pre-1979 period: \( \phi_p = 72.01 \) (See Ireland Table 1). Our empirical results, however, are qualitatively similar if we use the estimate for the post-1979 period (\( \phi_p = 77.10 \)).

Our main interest here is to study the propagation of nominal shocks. It will be useful, however, to introduce TFP shocks. For these shocks, we follow King and Rebelo (1999) and set \( \rho_z = 0.979 \) and \( \sigma_z = 0.0072. \) In addition, we use quarterly data on \( M2 \) from 1959:1 to 2000:1 to estimate \( \rho_\mu = 0.69 \) and \( \sigma_\mu = 0.006. \)

Finally, the values of both \( \sigma \) and \( \nu \) remain to be set. Prescott (1986) argues for a value of \( \sigma \) between 1 and 2, and Chari, Kehoe, and McGrattan (2000) set \( \nu \) so that the model implies a relative volatility of investment that matches that observed in the data. Several combinations of \( \sigma \) and \( \nu \) satisfy these criteria and deliver similar empirical results. As a benchmark, we set \( \nu = 0 \) and \( \sigma = 1.5 \) to ensure that the standard deviation of investment is 2.9 times the standard deviation of output, as in our post-war US sample. In the alternative models presented below, we keep \( \sigma = 1.5 \) and vary \( \nu \) to match the relative volatility of investment.

2.7 Empirical Results

The last two columns of Table 2 show the first autocorrelation of output and inflation, while Figure 1 displays the autocorrelations of output and inflation for up to 20 lags. The
autocorrelations in the post-war US data shown in all tables and figures are computed as
the sample autocorrelations of output and inflation over the full 1959:1 to 2000:1 period. In
addition, the tables report the first autocorrelation of output and inflation for a subsample
of 1985:1 to 2000:1 (see Appendix A for a description of the data). Output corresponds to
the detrended logarithm of per capita gross domestic product and inflation to the detrended
first difference of the logarithm of the consumer price index. In the data, output displays an
upward trend. Following Chari, Kehoe, and McGrattan (2000), we detrend all variables
by removing a linear-quadratic trend. Although inflation does not possess a trend, we
nevertheless remove one. We do so to remove low frequency fluctuations in inflation. As
documented in Boileau and Letendre (2003), post-war US inflation is on average much
higher during the 1970s and early 1980s than during the 1960s and 1990s. This feature
alone would make inflation fluctuations extremely persistent. It is doubtful, however,
that it reflects a business cycle fluctuation of inflation. Our detrending method may not
completely eliminate the influence of this period, but it is a step in the right direction.

The first sample autocorrelations for the full post-war US sample are 0.97 for output
and 0.78 for inflation. At higher lags, the sample autocorrelations of output and inflation
decline slowly. The autocorrelations of output are positive for the first 18 lags, while
those of inflation are positive for the first 11 lags. Although the subsample truncation
will be mostly useful for the inventory models, note that output and inflation appear less
persistent for the more recent subsample. The subsample first autocorrelations decline to
0.93 for output and 0.42 for inflation.

The autocorrelations predicted by all models are computed as the average autocorre-
lations over 1000 simulations of 164 quarters (the number of quarters of the full post-war
US sample) using variables detrended as in the post-war US sample. Also, we evaluate
whether the moments predicted by the model are close to those observed in the full post-
war US sample using the methodology proposed by Gregory and Smith (1991). To do
so, we construct a confidence interval using the quantiles of the empirical distribution of
the predicted moment, and ask whether the confidence interval includes the corresponding
observed moment. In Table 2, we use the symbol † (††) to indicate that a 90 percent (95
percent) confidence interval of the simulated moment includes the corresponding observed
moment. Note that our evaluation is based on matching the higher full sample autocorrelations, which is more demanding because the models underpredict these autocorrelations.

Table 2 and Figure 1 show that the baseline model numerically and statistically underpredicts the persistence of output and inflation. The first-order autocorrelations of output and inflation predicted by the benchmark baseline model are only 0.15 and 0.03, and much smaller than those computed with either the full sample or the subsample. Figure 1 shows that the autocorrelations of output and inflation predicted by the benchmark baseline model also decline much more rapidly than those computed from the full US sample. In particular, the predicted autocorrelations of output are smaller than the observed autocorrelations for the first 16 lags. The predicted autocorrelations of inflation are smaller than their observed counterparts for the first 11 lags.

Table 2 also reports empirical results for our baseline model with money growth shocks only and TFP shocks only, while Figure 1 also reports autocorrelations for the baseline model with money shocks only. The autocorrelations predicted by these variants of the baseline model suggest that the persistence of exogenous TFP shocks has an overall negligible impact on the first-order autocorrelation of output in the Benchmark parametrization. In particular, the predicted first-order autocorrelation of output is 0.13 in the money shocks only model and 0.85 in the TFP shocks only model. Also, both variants of the baseline model produce higher autocorrelations of inflation than the benchmark version. Note however that the money shocks only variant requires a non-zero adjustment cost parameter to maintain the volatility of investment ($\nu = 0.6$). The money shocks only version without adjustment costs generates small autocorrelations for output and inflation and a large volatility of investment. The TFP shocks only version does not require this change.

As we are mainly concerned with the propagation of nominal shocks, Figure 2 displays the dynamic responses to a money growth shock computed from the benchmark baseline model. It shows the responses of output, inflation, and money growth in percent deviations from their steady-state levels. The responses document that the baseline model generates (large) real effects from the money growth shock. This occurs because firms find it costly to change nominal prices. The mechanism works as follows. The higher money growth generates a larger transfer from the monetary authority to the consumer. As long as
prices are sticky, the larger transfer raises the consumer’s real balances. The increase in real balances stimulates the consumer’s demand for the aggregate good, because it raises his wealth and because real balances and consumption are complements. The increase in the demand for the aggregate good raises the demand for all individual goods.

In reaction to the increase in the demand for its good, a monopolistic producer can change its price and output levels. The larger the change in price, the smaller the change in output required to meet the new demand. The relative sizes of the price and output changes depend on the cost of changing nominal prices and the marginal cost of production. The cost of changing nominal prices depends on $\phi_p$: the larger $\phi_p$, the more costly it is to raise prices. The marginal cost of production is 
\[
[1/\alpha]^\alpha[1/(1 - \alpha)]^{1-\alpha}[1/z(h^t)]w(h^t)^\alpha r^k(h^t)^{1-\alpha}.
\]
In equilibrium, the marginal cost is increasing in output. That is, raising output requires an increase in the demand for inputs, which pushes wages and rental rates up and raises the marginal cost.

If prices are not costly to change ($\phi_p = 0$), a producer meets the new demand by increasing its price, and no output response is necessary. If prices are costly to change ($\phi_p > 0$) while output is not (the marginal cost is constant), a producer meets the new demand by raising output, and no price response is necessary. As shown in Figure 2, a producer trades off the two costs and raises both its price and its output to meet the new demand.

The persistence of the changes in price and output also depends on the cost of changing nominal prices. As documented in Boileau and Letendre (2003), fluctuations in output and inflation become more persistent with larger values for $\phi_p$. However, Figures 1 and 2 show that, even with our calibrated large value for $\phi_p$, a monetary shock does not have long-lasting effect on output and inflation in the baseline model.

3. The Linear-Quadratic Model

The linear-quadratic model adds inventories to the baseline model. For this addition, we borrow several features from West (1990). In particular, producers face quadratic costs of changing the level of production and of deviating from a target ratio of sales
to inventories. Our version of the linear-quadratic model, however, differs from that of West. Our producers are monopolistic competitors that produce goods with both labor and capital, while his producer is a monopolist that produces goods with labor only. Also, our demand shocks are money growth shocks, while his are taste shocks.

Our version of the linear-quadratic model uses the consumer, the retailer, and the monetary authority of the baseline model.

3.1 Producers

Producer $i$ chooses labor, capital, inventories, and prices to maximize expected discounted profits

$$
\sum_{t=0}^{\infty} \sum_{h^t} q(h^t) \left( p_i(h^t) s_i^d(h^t) - P(h^t) \left[ w(h^t) l_i(h^t) + r^k(h^t) k_i(h^t) \right] \right),
$$

subject to the production technology in equation (9), the definition of net output in equation (10), the demand for good $i$ depicted in equation (6), and the definition of labor usage $l_i$. Following West (1990), labor usage is

$$
l_i(h^t) = n_i(h^t) + \frac{\zeta_1}{2} \left[ \Delta y_i(h^t) \right]^2 + \frac{\zeta_2}{2} \left[ x_i(h^{t-1}) - \eta s_i(h^t) \right]^2,
$$

where $x_i$ is the stock of inventories and $\Delta$ is the difference operator: $\Delta y_i(h^t) = y_i(h^t) - y_i(h^{t-1})$. Labor is used in three activities. The first term on the right side of equation (16) represents the time allocated to production. The second term reflects the labor used to change the level of production. Finally, the last term is a labor cost due to deviations of inventories from a fraction of sales. This term represents the labor cost associated with stockouts and is often called the convenience yield.

Finally, inventories evolve as

$$
x_i(h^t) = x_i(h^{t-1}) + y_i^n(h^t) - s_i(h^t).
$$

3.2 Market Clearing and Aggregation

In our symmetric equilibrium, clearing of the bond, capital, and labor capital markets are described by equations (14.1), (14.2), and

$$
N(h^t) = n(h^t) + \frac{\zeta_1}{2} \left[ \Delta Y(h^t) \right]^2 + \frac{\zeta_2}{2} \left[ X(h^{t-1}) - \eta S(h^t) \right]^2,
$$
where aggregate quantities are as before, except for \( n(h^t) = \int n_i(h^t)di = n_i(h^t) \) and \( X(h^t) = \int x_i(h^t)di = x_i(h^t) \). Clearing of the goods market requires equation (14.4), (14.5), (14.6), and

\[
X(h^t) = X(h^{t-1}) - S(h^t) + Y^n(h^t). 
\]  

(18.2)

### 3.3 Benchmark Parameter Values

Table 1 also reports the benchmark parameter values for the linear-quadratic model. The values are set similarly to those of the baseline model. The linear-quadratic model has three additional parameters: \( \zeta_1 \), \( \zeta_2 \), and \( \eta \). West (1990) estimates a cost function similar to that in equation (16). Although the exact specification differs, West’s estimates offer a good benchmark (see West Table III). He provides estimates for \( \zeta_1/2 \), \( \zeta_2/2 \), and \( \eta \). Estimates for \( \zeta_1/2 \) range from 0.344 to 0.366 and estimates for \( \zeta_2/2 \) range from 0.111 to 0.145. Accordingly, we set \( \zeta_1 = 0.7 \) and \( \zeta_2 = 0.25 \). West also provides estimates for \( \eta \) that range between \(-0.040\) and \(-0.057\), but argues that a value between 0.4 and 0.7 reflects the general consensus. We set \( \eta = 0.68 \) so that steady-state sales are 60 percent of available goods (output plus inventories) as in the full post-war US sample.

### 3.4 Empirical Results

The empirical results on the persistence of output and inflation appear in Table 3 and Figure 3. Table 3 also reports the relative volatility of sales to output, the relative volatility of changes in inventories to output, and the correlation between changes in inventories and output. As for the autocorrelations, these moments are computed from the US sample and the model. In the post-war US sample, the relative volatility of sales is the ratio of the standard deviation of the logarithm of per capita sales to the standard deviation of the logarithm of per capita gross domestic product. The relative volatility of inventories is the ratio of the standard deviation of changes in inventories to the standard deviation of the logarithm of output. Changes in inventories correspond to the ratio of changes in private per capita inventories to per capita gross domestic product.

The volatility of sales and inventory investment and the correlation between inventory investment and output computed from the full sample appear different from those
computed in the subsample. For the full sample, we find the standard facts: sales are less volatile than output and inventories are highly procyclical. For the subsample, however, sales are more volatile than output and inventories are almost twice as volatile as in the full sample. In addition, the procyclicality of inventories is diminished: the correlation between inventories and output is less than half that of the full sample. These changes in the post mid-1980s behavior of sales and inventories are discussed in Kahn, McConnell, and Perez-Quiros (2001) and McConnell and Perez-Quiros (2000). Finally, recall that our evaluation is based on matching the full sample moments. As will become evident, this is more demanding for the inventory models than matching the subsample moments.

The predicted autocorrelations suggest that adding inventories raises the persistence of output and inflation. The first-order autocorrelation of output predicted by the benchmark linear-quadratic model is 0.91. This is much larger than the autocorrelation of 0.15 produced by the benchmark baseline model. It is also not statistically different at the 5 percent level from the autocorrelation of 0.97 observed in the full post-war US data. The first-order autocorrelation of inflation predicted by the benchmark linear-quadratic model is 0.49. Although this value is much larger than the 0.03 predicted by the benchmark baseline model, it is still much smaller than the value of 0.78 observed in the full post-war US sample. As shown in Figure 3, the predicted autocorrelations of output decline slowly, and are positive for all, but the last few displayed lags. Unfortunately, the predicted autocorrelations of inflation decline rapidly, and are positive only for the first three lags.

The results for the money shocks only and TFP shocks only experiments suggest that the persistence of exogenous TFP shocks is partially responsible for the added persistence of output, but not for the added persistence of inflation. In the money shocks only version, the predicted first-order autocorrelation of output drops to 0.76, and is now statistically different from its observed counterpart. The predicted first-order autocorrelation of inflation rises slightly to 0.52. Also, the predicted higher-order autocorrelations of output decline more rapidly, and are positive only for the first four lags. The autocorrelations of inflation also decline rapidly, and are positive for the first three lags.

The linear-quadratic model fails to replicate some standard inventory facts. Even though the model is consistent with the fact that changes in inventories are less volatile than
output, sales are more volatile than output and changes in inventories are countercyclical. Note that these moments are not independent. If inventories are countercyclical, output is raised and inventories are depleted to meet an increase in sales. The result is that sales are more volatile than output. If inventories are procyclical, output is raised more than sales, such that sales are less volatile than output (see equation 18.2). The simulation results suggest that money growth shocks promote countercyclical changes in inventories. The TFP shocks only version produces a small negative correlation between changes in inventories and output.\footnote{Note that this result is not robust, and that TFP shocks often promote procyclical inventories. For example, setting \( \zeta_1 = 0.01 \) and \( \zeta_2 = 10 \) generates a small positive correlation of 0.05.}

Note that the linear-quadratic model matches fairly well the moments in the recent subsample. In particular, the predicted moments closely match the autocorrelations of output and inflation, and reproduce the ranking of the volatility of sales, changes in inventories, and output. The model, however, generates modestly countercyclical changes in inventories, whereas the subsample shows modestly procyclical changes in inventories.

Figure 4 displays the dynamic responses to a money growth shock for the benchmark linear-quadratic model. The responses of output and inflation predicted by the benchmark linear-quadratic model are more persistent than those predicted by the baseline model. The higher persistence of output predicted by the linear-quadratic model is attributable to the fact that producers can vary inventories to meet the new demand. In the baseline model, a producer meets a larger demand by increasing price and output. In making his decisions, he accounts for the cost of adjusting prices and for the (increasing) marginal cost of production. In the linear-quadratic model, a producer meets a larger demand by increasing price and output, and by depleting inventories. He must account for the cost of adjusting prices, and the marginal cost of production, as well as for the cost of changing output and the cost of having inventories deviate from a fraction of sales. With the benchmark parameter values, a producer adjusts price, output, and inventories to trade off all these costs. Note that the reduction in inventories ensures that output does not increase as much as in the baseline model. Because of the increasing marginal cost of

\footnote{Note that this result is not robust, and that TFP shocks often promote procyclical inventories. For example, setting \( \zeta_1 = 0.01 \) and \( \zeta_2 = 10 \) generates a small positive correlation of 0.05.}
production, the reduction in the stock of inventories also ensures that the change in output is lasting to gradually replenish inventories. Overall, these responses are consistent with those presented in West (1990).

We wish to verify the robustness of these results to the values of the additional parameters $\zeta_1$, $\zeta_2$, and $\eta$. To that end, we perform three experiments on the benchmark linear-quadratic model.

Our first experiment investigates the effects of the cost of changing production. This cost offers a production smoothing motive that may explain the increase in the persistence of output. For this experiment, we reduce this cost by lowering $\zeta_1$ from 0.7 to 0.01. The results of this experiment appear as Low Smoothing. Diminishing the cost of changing output reduces the first-order autocorrelation of output to 0.80. This is larger than that predicted by the benchmark baseline model, but is statistically different from the autocorrelation observed in post-war US data at the 5 percent level. Clearly, the gradual adjustment of inventories adds to the persistence of output fluctuations. Otherwise, diminishing the cost of changing output has little effects. The first-order autocorrelation of inflation is still too small, sales are still more volatile than output, and changes in inventories are still countercyclical.

Our second experiment investigates the effects of the cost of having inventories deviate from a fraction of sales (the convenience yield cost). For this experiment, we make the deviations more costly by raising $\zeta_2$ from 0.25 to 4. The results appear as High Yield Costs. Raising this cost marginally improves the behavior of inventories and sales: changes in inventories are less countercyclical and both changes in inventories and sales are less volatile. Raising this cost, however, severely reduces the persistence of inflation.

Our last experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. A large steady-state level of this ratio is associated with a low convenience yield of having inventories and a low steady-state level of inventories. For this experiment, we raise the steady-state ratio of sales to available goods from 0.6 to 0.82 by reducing $\eta$ from 0.68 to 0.24. This value for the ratio of sales to available goods is similar to that obtained in Bils and Kahn (2000). The results of this experiment appear as Low Convenience. The increase in the steady-state ratio of sales to available goods has very
little impact on the autocorrelations of output and inflation, as well as on moments of sales and changes in inventories.

4. The Factor of Production Model

The factor of production model adds inventories to the baseline model by following Kydland and Prescott (1982) and Christiano (1988). In particular, inventories are an input in production, because they reduce down time and help economize on labor. Our version of the factor of production model is different from that of Kydland and Prescott. Importantly, our producers are monopolistic competitors, while theirs are perfect competitors. Also, we consider both technology and monetary growth shocks, while they consider only technology shocks.

Our version of the factor of production model retains the consumer, the retailer, and the monetary authority of the baseline model.

4.1 Producers

Monopolistic producer $i$ chooses labor, capital, inventories, and prices to maximize expected discounted profits given by equation (8) subject to the production technology in equation (9), the definition of net output in equation (10), the demand for good $i$ in equation (6), and the evolution of inventories in equation (17). In this case, however, gross output of good $i$ is produced using

$$y_i(h_t) = z(h_t)\left(\left[(1 - \ell)(h_t)^{-\varepsilon} + \ell x_i(h_t^{t-1})^{-\varepsilon}\right]^{-1/\varepsilon}\right)^{\alpha} n_i(h_t)^{1-\alpha}, \quad (19)$$

where $1/(1 + \varepsilon)$ is the elasticity of substitution between capital and inventories.

4.2 Market Clearing and Aggregation

In our symmetric equilibrium, the bond, capital, labor, and goods markets clear as in equations (14.1), (14.2), (14.3), (14.4), (14.5), (14.6), and (18.2). Aggregate quantities are as in the linear-quadratic model, except for employment which is defined as in the baseline model.
4.3 Benchmark Parameter Values

Table 1 reports the benchmark parameter values. The values are similar to those of the previous models. The factor of production model has two new parameters: $\varepsilon$ and $\ell$. Kydland and Prescott (1982) set $\varepsilon = 4$ and $\ell = 0.28 \times 10^{-5}$ to ensure that the elasticity of substitution between capital and inventories is low and that inventories represent about one-fourth of output. Following these guidelines, we set $\varepsilon = 4$ and $\ell = 6 \times 10^{-7}$ so that the elasticity is low and that steady-state sales are 60 percent of available goods.

4.4 Empirical Results

The empirical results appear in Table 4 and Figure 5. The predicted autocorrelations suggest that having inventories as an input raises the persistence of output as much as in the linear-quadratic model, and raises the persistence of inflation beyond that predicted by both the baseline and linear-quadratic models. The first-order autocorrelation of output predicted by the benchmark version is 0.91 and is not statistically different from its observed counterpart at the 5 percent level. The first-order autocorrelation of inflation predicted by the benchmark version is 0.62 and is statistically different from its observed counterpart at the 5 percent level. In additions, the predicted autocorrelations of output are positive for the first 17 lags, while those of inflation are positive for the first four lags.

Otherwise, the factor of production model behaves similarly to the linear-quadratic model. The predicted autocorrelations also suggest that the persistence of exogenous TFP shocks is partially responsible for the added persistence of output, but not for the added persistence of inflation. The predicted moments of changes in inventories and sales indicate that changes in inventories are less volatile than output, that sales are more volatile than output, and that changes in inventories are countercyclical. Finally, money growth shocks promote countercyclical changes in inventories, while TFP shocks promote procyclical changes in inventories.

The moments predicted by the factor of production model match well the moments in the recent subsample. The model predicts persistent output and inflation fluctuations, and predicts the subsample ranking of the volatility of sales, changes in inventories, and output. The model, however, generates modestly countercyclical changes in inventories,
whereas the subsample show modestly procyclical changes in inventories.

Figure 6 displays the dynamic responses to a money growth shock for the benchmark factor of production model. As for the linear-quadratic model, the dynamic responses of the factor of production model differ from that of the baseline model because producers can vary inventories to respond to changes in demand. In the factor of production model, a producer meets a larger demand by increasing price and output, and by depleting inventories. In making his decisions, he accounts for the cost of adjusting prices and the increasing marginal cost of production. In this case, the short-run marginal cost of production depends on inventories. A reduction of inventories, however, is not very costly in terms of lost output, because inventories play only a minor role in production. As in the linear-quadratic model, the depletion of inventories requires lasting output increases to gradually replenish inventories.

We wish to verify the robustness of these results to the values of the additional parameters $\varepsilon$ and $\ell$. For this, we perform two experiments on the factor of production model.

Our first experiment investigates the effects of the elasticity of substitution between capital and inventories. A reduction of this elasticity forces capital and inventories to be less substitutable. For our experiment, we reduce the elasticity by raising $\varepsilon$ from 4 to 10. The results of this experiment appear as Low Elasticity. Reducing the elasticity has no impact on the correlation between changes in inventories and output, but reduces the relative volatility of sales. The lower elasticity also marginally reduces the autocorrelations of output and inflation, but these are still larger than those produced by the baseline model.

Our second experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. As in the linear-quadratic model, a large steady-state level of this ratio is associated with a low convenience yield from inventories and a low steady-state level of inventories. For our experiment, we raise the steady-state ratio of sales to available goods from 0.60 to 0.82 by lowering $\ell$ from $6 \times 10^{-7}$ to $2.25 \times 10^{-9}$. The results appear as Low Convenience. Reducing the steady-state level of inventories slightly diminishes the autocorrelations of output and inflation. Changes in inventories become marginally more countercyclical, and the relative volatilities of sales and changes in inventories are reduced.
5. The Shopping-Cost Model

The shopping-cost model adds inventories to the baseline model by adopting some elements of Bils and Kahn (2000). In particular, producers face a demand that depends on the available stock of goods. That is, consumers, via retailers, find it costly to engage in shopping activities. A larger stock of available goods helps economize on the resources expanded while shopping. Our shopping-cost model, however, differs from that of Bils and Kahn. Our demand for goods is derived from the consumer’s problem, while their demand is a reduced form. Finally, our demand shocks are money growth shocks, while theirs are real demand shocks.

Our version of the shopping model uses the consumer and the monetary authority of the baseline model.

5.1 The Retailer

The competitive retailer chooses purchases to maximize profits given in equation (4) subject to the aggregation technology displayed in equation (5). In this case, however, the retailer finds it costly to purchase goods. The cost of purchasing \( s_i(h^t) \) units of good \( i \) is \( [1 - \gamma a_i(h^t) \xi] s_i(h^t) \), such that

\[
g_i(h^t) = \gamma a_i(h^t) \xi s_i(h^t),
\]

where \( a_i(h^t) = y^n_i(h^t) + x_i(h^{t-1}) \) is the stock of good \( i \) available.

The retailer’s first-order conditions imply the goods demand function

\[
s_i^d(h^t) = \left[ \frac{p_i(h^t)}{P(h^t)} \right]^{\theta} G(h^t) \left[ \gamma a_i(h^t) \xi \right]^{\theta - 1}.
\]

Finally, the demand for all goods combined with the zero-profit condition of the retailer yields the price index

\[
P(h^t) = \left( \int p_i(h^t)^{1-\theta} \left[ \gamma a_i(h^t) \xi \right]^{\theta - 1} \, di \right)^{1-\theta}.
\]
5.2 Producers

Producer $i$ chooses labor, capital, inventories, and prices to maximize expected discounted profits given in equation (8) subject to the production technology in equation (9), the definition of net output in equation (10), the demand for goods in equation (21), and the evolution of inventories in equation (17).

5.3 Market Clearing and Aggregation

In our symmetric equilibrium, the bond, capital, and labor markets clear as in equations (14.1), (14.2), and (14.3). Clearing of the goods market requires equations (14.4), (14.5), (18.2), and

$$G(h_t) = S(h_t)\gamma A(h_t)^\xi.$$  

Aggregate quantities are as in the factor of production model, with the addition of $A(h_t) = \int a_i(h^t)di = a_i(h^t)$.

5.4 Benchmark Parameter Values

Table 1 reports the benchmark parameter values. The shopping-cost model has two new parameters: $\gamma$ and $\xi$. Although the models differ, the parameter estimates of Bils and Kahn (2000) offer a good benchmark. They provide estimates for $\xi(\theta - 1)$ (See Bils and Kahn Table 6). The constrained estimates range from 0.023 to 0.486. As in our previous models, we set $\xi = 0.0168$ so that steady-state sales are 60 percent of available goods. Given our value of $\theta = 10$, the implied value of $\xi(\theta - 1)$ is 0.151, which is well within Bils and Kahn's range of estimates. Finally, we set $\gamma = 0.9906$ to remove steady-state transaction costs.

5.5 Empirical Results

The empirical results appear in Table 5 and Figure 7. These results suggest that the shopping-cost model best explains the persistence of output and inflation in the full sample. The autocorrelations of output and inflation predicted by the benchmark shopping-cost model are larger than those predicted by our previous inventory models. The predicted
first-order autocorrelations of output and inflation are 0.92 and 0.71. These values are not statistically different from the values observed in the full post-war US sample at the 10 percent level. Also, the predicted autocorrelations of output decline slowly, and are positive for all but the last few displayed lags. The predicted autocorrelations of inflation decline too rapidly, but are positive for the first six lags.

Interestingly, the persistence of exogenous TFP shocks plays only a marginal role in the persistence of output and inflation, but a large role in the moments of sales and inventories. The autocorrelations of output and inflation predicted by the money shocks only version are similar to those predicted by the benchmark version.

In addition, Table 5 documents that the benchmark shopping-cost model produces procyclical changes in inventories. The correlation between changes in inventories and output, however, is only 0.07, while that observed in the full post-war US sample is 0.50. Finally, as in the previous inventory models, money growth shocks promote countercyclical changes in inventories, while TFP shocks promote procyclical changes in inventories.

Overall, the moments predicted by the shopping-cost model match the data better in the subsample. The model predicts persistent output and inflation fluctuations and modestly procyclical changes in inventories. It predicts the subsample ranking of the volatility of sales and output, but generates as much volatility for changes in inventories as it does for output.

Figure 8 displays the dynamic responses to a money growth shock for the benchmark shopping-cost model. As for the previous inventory models, the dynamic responses of the shopping-cost model differ from that of the baseline model because producers can vary inventories to respond to changes in demand. In the shopping-cost model, a producer meets a larger demand by increasing price, increasing output, and depleting inventories. In making his decisions, he accounts for the cost of adjusting prices and the increasing marginal cost of production, as well as the impact of his output and inventory decisions on sales (e.g. an increase in output increases the stock of available goods and makes shopping less difficult). With the benchmark parameter values, the producer raises both output and prices, and depletes inventories to meet the new demand. The extra lever provided by the impact of the stock of available goods on the demand for goods has two implications. First,
sales do not respond as much as in the previous inventory models. Second, output responds much less than in the previous inventory models. That is, over time, the producer gradually replenishes its inventories and manages the demand by smoothly increasing output.

We wish to verify the robustness of these results to the values of the additional parameters \( \gamma \) and \( \xi \). To that end, we perform some experiments on the benchmark shopping-cost model.

Our first experiment investigates the effects of the steady-state level of shopping costs. To do so, we set \( \gamma \) to 0.9415 so that five percent of goods are lost during shopping in the steady state. The results of this experiment appear as High Shopping Costs. The results document that the steady-state level of these costs has no effects on the persistence of output and inflation or on the moments of sales and inventories.

Our second experiment investigates the effects of the steady-state level of the ratio of sales to all available goods. As before, a large steady-state level of this ratio is associated with a low convenience yield from inventories and a low steady-state level of inventories. For our experiment, we raise the steady-state ratio of sales to available goods from 0.60 to 0.82 by reducing \( \xi \) from 0.0168 to 0.0123. The results of this experiment appear under Low Convenience. Reducing the steady-state level of inventories has no impact on the autocorrelations of output and inflation. Changes in inventories become somewhat less volatile and procyclical.

Our last experiment also investigates the effects of the steady-state level of the ratio of sales to all available goods. Instead of increasing the steady-state ratio, however, we lower it to 0.187 by raising \( \xi \) to 0.054. Note that the resulting value of \( \xi (\theta - 1) \) is 0.486, the upper boundary of the range of estimates presented in Bils and Kahn (2000). The results appear under High Convenience. As before, changing the steady state level of inventories has little impact on the persistence of output and inflation, but makes changes in inventories more procyclical and reduces the volatility of sales. In fact, the volatility of sales relative to output and the correlation between output and changes in inventories are not statistically different from their observed counterparts at the 5 percent level.
6. Conclusion

Postwar US business cycle fluctuations of output and inflation are remarkably persistent. Standard sticky-price monetary business cycle models with explicit microfoundations, however, fail to explain this persistence. Our objective is to determine whether adding inventories to a standard sticky-price monetary business cycle model raises the predicted persistence of output and inflation.

To fulfill this objective, we compare the persistence of output and inflation computed from three different models with inventories to the persistence computed in a model without inventories. Our three models with inventories are a linear-quadratic model, a factor of production model, and a shopping-cost model. These models emphasize different roles for inventories. In the linear-quadratic model, producers manage inventories to avoid the costs associated with changing output and with having inventories deviate from a target fraction of sales. In the factor of production model, producers manage a stock of inventories that is an input in production. Finally, in the shopping-cost model, producers manage inventories that affect the demand for its goods by making it easier for consumers to shop.

We find that the propagation properties of inventories depend partly on the role played by inventories. In all models, we find that adding inventories raises the persistence of output and inflation. Adding inventories as in the linear-quadratic model or as in the factor of production model raises the persistence of output sufficiently to match the persistence of output in US data for the period 1959:1 to 2000:1 (the ‘full US sample’). These two models, however, are unable to produce fluctuations in inflation that are as persistent as those in the full US sample. Adding inventories as in the shopping-cost model raises the persistence of both output and inflation sufficiently to allow the model to match the persistence of these variables in the full US sample. All three models have some shortcomings stemming from the behavior of sales and inventories along the business cycle.

In the full US sample, the standard deviation of sales is smaller than the standard deviation of output while the correlation between changes in inventories and output is positive and large. Unfortunately, none of the inventory models can produce this ranking
of standard deviations of sales and output as well as the large and positive correlation between output and changes in inventories.

The models with inventories are better able to match the properties of the data when we focus on the subsample 1985:1-2000:1. All of the models are able to generate fluctuations in output and inflation that are as persistent (or more persistent) than those in the US subsample. All models with inventories are consistent with the fact that sales are more volatile than output in the US subsample. However, even if we restrict our attention to the shorter sample, none of the models match all five moments reported in the tables. The shopping-cost model counterfactually predicts that changes in inventories are as volatile as output whereas the linear-quadratic and factor of production models predict counterfactually countercyclical changes in inventories.
Appendix A — Data Appendix

Our quarterly post-war US sample covers the 1959:1 to 2000:1 period. It comprises the following: Gross Domestic Product: Bureau of Economic Analysis, NIPA Table 1.2; Change in Private Inventories: Bureau of Economic Analysis, NIPA Tables 1.2, 5.11A, 5.11B; Private Inventories: Bureau of Economic Analysis, NIPA Tables 5.13A, 5.13B; Final Sales of Domestic Business: Bureau of Economic Analysis, NIPA Tables 5.13A, 5.13B; Consumer Price Index: Bureau of Economic Analysis, NIPA Table 7.1; Investment: fixed investment, Citibase, mnemonic GIFQF; Population: Citibase, mnemonic P16; and M2 Money Stock: FRED.

We construct per capita output $Y_t$ and per capita inventories $X_t$ by dividing Gross Domestic Product and Private Inventories by Population. Our measure of the price index $P_t$ is the Consumer Price Index. Finally, we construct quarterly per capita M2 data by averaging the monthly data and dividing by Population.
References


Table 1: Benchmark Parameter Values

**The Baseline Model**

Consumers  
$\beta = 0.99, \sigma = 1.5, \omega = 0.94, \chi = 0.39, \psi = 1.7119,$  
$\delta = 0.025, \nu = 0$

Producers  
$\alpha = 0.36, \phi_p = 72.01, \bar{z} = 1, \rho_z = 0.979, \sigma_z = 0.0072$

Retailers  
$\theta = 10$

Monetary Authority  
$\bar{\mu} = 1, \rho_{\mu} = 0.69, \sigma_{\mu} = 0.006$

**The Linear-Quadratic Model**

Consumers  
$\beta = 0.99, \sigma = 1.5, \omega = 0.94, \chi = 0.39, \psi = 1.6968,$  
$\delta = 0.025, \nu = 6.66$

Producers  
$\alpha = 0.36, \phi_p = 72.01, \zeta_1 = 0.7, \zeta_2 = 0.25, \eta = 0.68,$  
$\bar{z} = 1, \rho_z = 0.979, \sigma_z = 0.0072$

Retailers  
$\theta = 10$

Monetary Authority  
$\bar{\mu} = 1, \rho_{\mu} = 0.69, \sigma_{\mu} = 0.006$

**The Factor of Production Model**

Consumers  
$\beta = 0.99, \sigma = 1.5, \omega = 0.94, \chi = 0.39, \psi = 1.7022,$  
$\delta = 0.025, \nu = 9.6$

Producers  
$\alpha = 0.36, \phi_p = 72.01, \ell = 6 \times 10^{-7}, \varepsilon = 4,$  
$\bar{z} = 1, \rho_z = 0.979, \sigma_z = 0.0072$

Retailers  
$\theta = 10$

Monetary Authority  
$\bar{\mu} = 1, \rho_{\mu} = 0.69, \sigma_{\mu} = 0.006$

**The Shopping-Cost Model**

Consumers  
$\beta = 0.99, \sigma = 1.5, \omega = 0.94, \chi = 0.39, \psi = 1.7345,$  
$\delta = 0.025, \nu = 13.7$

Producers  
$\alpha = 0.36, \phi_p = 72.01, \bar{z} = 1, \rho_z = 0.979, \sigma_z = 0.0072$

Retailers  
$\theta = 10, \gamma = 0.9906, \xi = 0.0168$

Monetary Authority  
$\bar{\mu} = 1, \rho_{\mu} = 0.69, \sigma_{\mu} = 0.006$

Note: Several parameters are set endogenously. The values for $\psi$ and $\nu$ ensure that hours worked are 30 percent of the time endowment in the steady state and that the ratio of the standard deviations of the logarithm of investment and the logarithm of output is 2.9. The values for $\eta, \ell,$ and $\xi$ are set so that sales are 60 percent of all available goods (output plus inventories) in the steady state. Finally, the value for $\gamma$ is set to eliminate steady-state transaction costs.
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<thead>
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<th>Volatility Relative to Output</th>
<th>Correlation with Output</th>
<th>First-Order Autocorrelation</th>
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<td>Inventories</td>
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<td>TFP Shocks Only</td>
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Note: Entries under volatility relative to output show the ratio of the standard deviation of a variable to the standard deviation of output (in percentages). Entries under correlation with output show the contemporaneous correlation with output. Entries under first-order autocorrelation show the first-order sample autocorrelation of the variable. All variables are detrended by removing a linear-quadratic trend. The simulated moments are computed as the average over 1000 replications of 164 periods. The benchmark parameter values are discussed in Section 2.6. The alternative parameter values retain the benchmark values with the following changes: Money Shocks Only ($\rho_z=0$ and $\sigma_z=0$) and TFP Shocks Only ($\rho_\mu=0$ and $\sigma_\mu=0$). The symbol † (‡) indicates that a 90 (95) percent confidence interval includes the moment calculated in US data (the confidence interval runs from the 0.05 (0.025) to the 0.95 (0.975) quantiles of the frequency distribution of the simulated moments).
Table 3. The Linear-Quadratic Model

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<td>Benchmark</td>
<td>1.22</td>
<td>0.55</td>
<td>-0.16</td>
</tr>
<tr>
<td>Money Shocks Only</td>
<td>1.41</td>
<td>0.71</td>
<td>-0.34</td>
</tr>
<tr>
<td>TFP Shocks Only</td>
<td>1.12</td>
<td>0.43</td>
<td>-0.07</td>
</tr>
<tr>
<td>Low Smoothing</td>
<td>1.20</td>
<td>0.41</td>
<td>-0.31</td>
</tr>
<tr>
<td>High Yield Costs</td>
<td>1.12</td>
<td>0.38</td>
<td>-0.14</td>
</tr>
<tr>
<td>Low Convenience</td>
<td>1.22</td>
<td>0.53</td>
<td>-0.18</td>
</tr>
</tbody>
</table>

Note: Entries under volatility relative to output show the ratio of the standard deviation of a variable to the standard deviation of output (in percentages). Entries under correlation with output show the contemporaneous correlation with output. Entries under first-order autocorrelation show the first-order sample autocorrelation of the variable. All variables are detrended by removing a linear-quadratic trend. The simulated moments are computed as the average over 1000 replications of 164 periods. The benchmark parameter values are discussed in Section 3.3. The alternative parameter values retain the benchmark values with the following changes: Money Shocks Only ($\rho_z=0$ and $\sigma_z=0$), TFP Shocks Only ($\rho_\mu=0$ and $\sigma_\mu=0$), Low Smoothing ($\zeta_1=0.01$), High Yield Costs ($\zeta_2=4$), and Low Convenience ($\eta=0.24$). The symbol $^\dagger$ ($^{\dagger}$) indicates that a 90 (95) percent confidence interval includes the moment calculated in US data (the confidence interval runs from the 0.05 (0.025) to the 0.95 (0.975) quantiles of the frequency distribution of the simulated moments).
Table 4. The Factor of Production Model

<table>
<thead>
<tr>
<th></th>
<th>Volatility Relative to Output</th>
<th>Correlation with Output</th>
<th>First-Order Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>Inventories</td>
<td>Inventories</td>
</tr>
<tr>
<td>Post-war US Data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1959:1–2000:1</td>
<td>0.94</td>
<td>0.13</td>
<td>0.50</td>
</tr>
<tr>
<td>1985:1–2000:1</td>
<td>1.07</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>The Factor of Production Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
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<td>0.64</td>
<td>-0.21</td>
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<tr>
<td>Money Shocks Only</td>
<td>1.68</td>
<td>0.90</td>
<td>-0.57</td>
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<td>TFP Shocks Only</td>
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<td>0.02</td>
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<td>Low Elasticity</td>
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<td>Low Convenience</td>
<td>1.25</td>
<td>0.54</td>
<td>-0.26</td>
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</tbody>
</table>

Note: Entries under volatility relative to output show the ratio of the standard deviation of a variable to the standard deviation of output (in percentages). Entries under correlation with output show the contemporaneous correlation with output. Entries under first-order autocorrelation show the first-order sample autocorrelation of the variable. All variables are detrended by removing a linear-quadratic trend. The simulated moments are computed as the average over 1000 replications of 164 periods. The benchmark parameter values are discussed in Section 4.3. The alternative parameter values retain the benchmark values with the following changes: Money Shocks Only \((\rho_z=0 \text{ and } \sigma_z=0)\), TFP Shocks Only \((\rho_u=0 \text{ and } \sigma_u=0)\), Low Elasticity \((\epsilon=10)\), and Low Convenience \((\epsilon=2.25\times10^{-9})\). The symbol \(\dagger\) indicates that a 90 (95) percent confidence interval includes the moment calculated in US data (the confidence interval runs from the 0.05 (0.025) to the 0.95 (0.975) quantiles of the frequency distribution of the simulated moments).
<table>
<thead>
<tr>
<th></th>
<th>Volatility Relative to Output</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>Inventories</td>
<td>Inventories</td>
</tr>
<tr>
<td><strong>Post-war US Data</strong></td>
<td></td>
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<tr>
<td>1959:1–2000:1</td>
<td>0.94</td>
<td>0.13</td>
<td>0.50</td>
</tr>
<tr>
<td>1985:1–2000:1</td>
<td>1.07</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>The Shopping-Cost Model</strong></td>
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<tr>
<td>Benchmark</td>
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<td>1.01</td>
<td>0.07</td>
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<tr>
<td>Money Shocks Only</td>
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<tr>
<td>High Shopping Costs</td>
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<td>0.07</td>
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<td>Low Convenience</td>
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<td>High Convenience</td>
<td>1.28$^\dagger$</td>
<td>1.06</td>
<td>0.23$^\dagger$</td>
</tr>
</tbody>
</table>

Note: Entries under volatility relative to output show the ratio of the standard deviation of a variable to the standard deviation of output (in percentages). Entries under correlation with output show the contemporaneous correlation with output. Entries under first-order autocorrelation show the first-order sample autocorrelation of the variable. All variables are detrended by removing a linear-quadratic trend. The simulated moments are computed as the average over 1000 replications of 164 periods. The benchmark parameter values are discussed in Section 5.4. The alternative parameter values retain the benchmark values with the following changes: Money Shocks Only ($\rho_s=0$ and $\sigma_s=0$), TFP Shocks Only ($\rho_s=0$ and $\sigma_s=0$), High Shopping Costs ($\gamma=0.9415$), Low Convenience ($\xi=0.0123$), and High Convenience ($\xi=0.054$). The symbol $^\dagger$ ($^{\dagger\dagger}$) indicates that a 90 (95) percent confidence interval includes the moment calculated in US data (the confidence interval runs from the 0.05 (0.025) to the 0.95 (0.975) quantiles of the frequency distribution of the simulated moments).
Figure 2: The Baseline Model

Responses to Money Growth Shock

Yield from Baseline State

Money Growth
Inflation
Output
Figure 4: The Linear-Quadratic Model
Responses to Money Growth Shock

% Deviation from Steady State

Quarters

Output
Sales
Inflation
Money Growth
Figure 5. The Factor of Production Model Auto-correlations of Output

Figure 5. The Factor of Production Model Auto-correlations of Inflation
Figure 6: The Factor of Production Model Response to Money Growth Shock.
Figure 8: The Shopping-Cost Model
Responses to Money Growth Shock