Income Disparity, Inequality Aversion, and the Design of the Health Care System

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Abstract

If society is averse to inequality and there is some income disparity, private health care insurance is sometimes socially better and sometimes worse than public health care. However, a third system—public health care with the option to purchase supplemental health care insurance—is always socially preferred to private health care insurance. It is not always socially preferred to public health care. If a public health care system is in place, the existence of a latent demand for supplemental insurance is a sufficient but not necessary condition for the supplemental insurance system to be socially preferred to public health care.

1. Introduction

The introduction of public health care was largely motivated by a desire to ameliorate the welfare effects of ill health. Since public health care is paid for by all but consumed only by those who are ill, it raises the utility of the unhealthy and reduces the utility of the healthy. This kind of redistribution raises social welfare if society is averse to inequality. However, public health care also reduces social welfare by overriding consumer sovereignty. In the absence of a public health care program, differences in incomes or preferences would cause people to choose different levels of health care. The “equal access” character of public health care prevents people from exercising this choice, reducing their utilities and the value of any non-paternalistic social welfare function. A formal model of this trade-off is presented here, and used to describe the welfare implications of allowing people to
supplement their public health care coverage with private health care insurance.\textsuperscript{1} It will be shown that this combination is always preferred to private health care insurance on its own, and might or might not be preferred to public health care alone.

A social aversion to inequality is introduced by assuming that the social welfare function is strictly concave. As well, each agent is assumed to be initially uncertain of his state of health. Hammond \cite{6} defines two kinds of non-paternalistic social welfare function that can be employed under these circumstances.\textsuperscript{2} The \textit{ex ante social welfare function} is a function of the expected utilities, while the \textit{ex post social welfare function} is the expectation of a function of the realized utilities. There are no objective grounds for choosing between the two. In the present case the choice between them reflects value judgements about two pivotal issues. Should the health care system be designed to allay the agents’ fears of future illness, or to ameliorate the effects of illness once it has occurred? Does inequality aversion mean that society is averse to unequal prospects or to unequal outcomes? The first responses to these questions involve expected utilities, suggesting the \textit{ex ante} criterion. The second responses involve realizations, leading one to the \textit{ex post} criterion. I favour the second response to each question, and I suspect that I am not alone. Accordingly the \textit{ex post} criterion is employed below.

It is also assumed that a public health care program cannot redistribute income. That is, it cannot provide everyone with the same access to health care but charge health care premiums that vary with income (as would be the case if it were financed through an income tax). This assumption impounds the welfare effects of pure income redistribution, so that any observed changes in welfare are entirely the result of the change in the allocation of health care.

There are, of course, environments in which the design of a public health care program affects the government’s ability to redistribute income. Nichols and Zeckhauser \cite{10} and Blackorby and Donaldson \cite{2} show that the provision of some private good (such as health care) can be used to relax the selection constraints when the government does not have perfect information about the attributes of agents. Likewise, Rochet \cite{12} and Cremer and Pestieau \cite{5} have shown that social insurance (health care is again one example) can increase the extent of redistribution if there is a negative correlation between income and the

\textsuperscript{1}Both the United Kingdom and Australia currently allow such insurance, and a recent Supreme Court ruling suggests that Canada might soon join them.  
\textsuperscript{2}This issue was addressed earlier by Starr \cite{13}, Mirrlees \cite{9} and Harris \cite{7}, but Hammond’s set-up is particularly useful for the problem at hand.
risks to health. Their arguments are extended by Petretto [11] and by Boadway, Leite-Monteiro, Marchand and Pestieau [3, 4]. Neither of these considerations are relevant to the current model, which assumes that the agents and the government are always equally well informed.

2. The Model

Consider an economy in which there is a continuum of agents. Half of the agents are rich ($R$) and half are poor ($P$). Each group of agents has measure 1. The income of the rich is $y_R$ and the income of the poor is $y_P$, where

$$y = y_R + y_P, 0 < y_P < y/2$$

Aggregate income $y$ is exogenous. All of the income is used to purchase consumption goods and medical care, with a unit of either good costing one unit of income.

Each agent is healthy with probability $1 - \pi$ and ill with probability $\pi$. The state of each person’s health is independent of every other person’s health. If a person is healthy, his utility is

$$U_i = u(c_i) \quad i = R, P$$

where $c_i$ is consumption and $u$ is a utility function. The function $u$ is strictly increasing, strictly concave, twice continuously differentiable, and has the properties

$$u'(0) = \infty, \lim_{c \to \infty} u'(c) = 0$$

If he is ill, his utility is

$$V_i = v(c_i, m_i) \quad i = R, P$$

where $m_i$ is the quantity of health care received. The function $v$ is strictly increasing, strictly concave, twice continuously differentiable and has the properties

$$v_1(0, m) = \infty, \lim_{c \to \infty} v_1(c, m) = 0$$

$$v_2(c, 0) = \infty, \lim_{m \to \infty} v_2(c, m) = 0$$

A person who is ill has lower utility than a person who is healthy:

$$u(c) \geq \lim_{m \to \infty} v(c, m) \quad \text{for all } c$$
It is also assumed that an increase in health care raises a sick person’s marginal utility of consumption, or at least does not reduce it.

\[ v_{12} \geq 0 \]

Several health care systems are described below. Under each system the health care of the agents in each group (the rich or the poor) is covered by an insurance program that is financed by a levy on the incomes of the agents in that group, implying

\[ c_i = y_i - \pi m_i \quad i = R, P \] \hspace{1cm} (3)

The health care system is not permitted to redistribute income between the two groups.

A social welfare function is used to evaluate the health care systems. Society is assumed to be averse to inequality, and this aversion is introduced by employing Atkinson’s [1] social welfare function. However, as noted above, two social welfare functions are still possible. The first is a function of the individual expected utilities:

\[ \tilde{W} = \sum_{i=R,P} [\pi V_i + (1 - \pi)U_i]^\alpha \quad 0 < \alpha < 1 \]

Hammond [6] calls this function the \textit{ex ante social welfare function}. The second is the expectation of a function of the realized utilities, which Hammond calls the \textit{ex post social welfare function}.

\[ W = \sum_{i=R,P} [\pi(V_i)^\alpha + (1 - \pi)(U_i)^\alpha] \quad 0 < \alpha < 1 \]

The distinction between ex ante and ex post welfare seldom arises because social welfare is generally assumed to be linear in the utilities (i.e., \( \alpha \) is set equal to 1) so that the expectations operator passes through. The necessity of choosing between them in the present case follows directly from the strict concavity of the social welfare function, or equivalently, from society’s aversion to inequality. No objective criterion governs this choice, but for the reasons already outlined, the ex post function is used below.

The composite function \( \tilde{W}(m_R, c_R, m_P, c_P) \) is obtained by substituting (1) and (2) into \( W \). Since \( W \) is a strictly concave function of \( U_i \) and \( V_i \) \((i = R, P)\), and \( U_i \) and \( V_i \) are strictly concave functions of \( c_i \) and \( m_i \), the composite function is also strictly concave.
3. A Benchmark: Two-Tier Public Health Care

Three health care systems are examined below: private health care insurance, one-tier public health care (in which the rich and the poor receive the same care and pay the same health care premiums), and one-tier public health care with the option to purchase supplemental private health care insurance. These systems can be modelled as deviations from a two-tier public health care system in which the rich and poor agents receive different care and pay premiums that reflect the care received.

Two-tier public health care provides a specified quantity of health care to the unhealthy agents in each group, and is funded by a lump-sum tax. The tax paid by each agent is equal to the expected cost of his health care. The optimal two-tier system is the list \((m_R^2, c_R^2, m_P^2, c_P^2)\) that maximizes \(W(m_R, c_R, m_P, c_P)\) subject to the budget constraints in (3) and a set of non-negativity constraints. The restrictions on \(u\) and \(v\) imply that the non-negativity constraints are not binding. The solution to this problem satisfies the budget constraints (3) and the conditions

\[
f(c_i, m_i) \equiv (1 - \pi)r(c_i, m_i)u'(c_i) + \pi v_1(c_i, m_i) - v_2(c_i, m_i) = 0 \quad i = R, P \tag{4}
\]

where

\[
r(c_i, m_i) \equiv \left[ \frac{v(c_i, m_i)}{u(c_i)} \right]^{\alpha} < 1 \quad \tag{5}
\]

Since the objective function is strictly concave and the constraints are linear, a solution exists and is unique.

The function \(r\) shows the relative value of increases in the utility of the healthy and the utility of the sick in each group under the social welfare function \(W\). Specifically, an increase in the utility of the healthy raises social welfare by only \(r\) times as much as an increase in the utility of the sick. Equation (4) states that the optimal level of consumption for each group occurs where the social benefit of a small increase in consumption is exactly offset by the cost of the health care that must be given up to obtain that increase. The benefit of an additional unit of consumption is given by the first two terms of \(f\). The increase in consumption benefits both the healthy, who have measure \(1 - \pi\), and the sick, who have measure \(\pi\). The gains of the healthy are weighted by the factor \(r\) because their gains have a smaller impact on social welfare than the gains of the sick. The third term of \(f\) shows the impact on the associated reduction in health care. The sick have measure \(\pi\) and each member of this group must give up \(1/\pi\) units of health care, so the decline in their aggregate utility is simply \(v_2\).
The strict concavity of the objective function and linearity of the constraints implies that the second-order conditions are necessarily satisfied, and these conditions imply that the locus $f = 0$ is upward sloping or, if downward sloping, flatter than the budget constraint. Figure 1 shows the two possibilities. (Since there exists a unique solution for every value of $y$, the locus can have downward sloping segments but it cannot be downward sloping everywhere.) The equilibrium on the left gives rise to the expected result that a planner will respond to an increase in income by raising both consumption and health care. By contrast, in the neighborhood of the equilibrium on the right, the planner responds to an increase in income by raising health care but reducing consumption. What determines the slope of the locus? The sign of the slope of the locus is the negative of the sign of the derivative

$$f_1 = (1 - \pi)r_1 u' + (1 - \pi)r u'' + \pi v_{11} - v_{12}$$

Inspection of this expression shows that the sign of the first term is indeterminate, while the remaining terms—the adjustments to the marginal utilities—are negative. No condition can be sufficient for a negative slope, but

$$r_1 = (1 - \alpha)r \left[ \frac{v_1}{v} - \frac{u'}{u} \right] \leq 0$$

is sufficient for a positive slope. That is, the locus is upward sloping if the elasticity of utility with respect to consumption is at least as great for the healthy as it is for sick. In a world of many goods, it would seem reasonable to assume that healthy people have a higher marginal utility of consumption than do sick people. (Illness reduces the pleasure that a person takes from any activity. People who are

\[\text{The second-order conditions are}\]

\[\frac{df(c_i, m_i)}{dc_i} < 0 \quad i = R, P\]

where $m$ is determined by (3), implying

\[\frac{dm_i}{dc_i} \bigg|_{f=0} = -\frac{f_1}{f_2} \bigg|_{f=0} > -\frac{1}{\pi}\]

where $f_2$ is positive and $f_1$ is ambiguous.

\[\text{If } r_1 \text{ is negative, a social planner can increase the ratio } v/u \text{ by increasing consumption. If } r_1 \text{ is non-negative, a social planner can increase this ratio only by providing more health care. The sufficient condition places the social planner in a black-and-white situation: his only tool for achieving greater equality is health care provision.}\]
ill also have circumscribed options; since they consume a reduced set of goods more intensively, their marginal utility of consumption drops more quickly.) However, the above inequality requires something stronger: that an increase in consumption causes the utility of the healthy to rise by an equal or greater proportion than that of the sick:

\[
u' \geq \left( \frac{u}{v} \right) v_1\]

This restriction is difficult to rationalize, and it is not necessarily satisfied by simple functional forms. For example, it is satisfied if

\[v(c_i, m_i) \equiv u(c_i)v(m_i)\]

but not if

\[v(c_i, m_i) \equiv u(c_i) - \psi(m_i)\]

Nevertheless, it will be assumed below that the locus \( f = 0 \) is everywhere upward sloping, either because the sufficient condition is satisfied or because the adjustments to the marginal utilities are large enough to offset the change in \( r \). Under this assumption, the rich have higher consumption than the poor and receive more health care if they become ill.
4. Private Health Care Insurance

Now suppose that health care is provided in competitive markets. Health care itself is assumed to be supplied by competitive firms at its marginal cost of production. However, the agents do not purchase health care directly. The agents purchase insurance from a group of competitive insurers, who subsequently finance the health care of those who become ill. The insurance contract purchased by each agent specifies the quantity of health care that he will receive if he becomes ill. Competition among insurers ensures that the health care premium is actuarially fair, that is, the cost of each unit of illness-contingent health care is \( \pi \). It follows that the combinations of consumption and health care that he can purchase are again given by (3).

The expected utility of an agent in group \( i \) (\( i = R, P \)) who purchases insurance is

\[
EU_i = (1 - \pi)u(c_i) + \pi v(c_i, m_i)
\]

His optimal commodity bundle is the pair \( (m_i^N, c_i^N) \) that maximizes his expected utility subject to the budget constraint in (3). The optimal commodity bundle is
unique and satisfies the condition

\[ g(c_i, m_i) \equiv (1 - \pi)u'(c_i) + \pi v_1(c_i, m_i) - v_2(c_i, m_i) = 0 \]

The characteristic of the optimal commodity bundle is that there is no small feasible adjustment that raises expected utility. Increasing consumption by one unit, for example, raises the agent’s utility by the marginal utility of consumption, which is \( u' \) with probability \( 1 - \pi \) and \( v_1 \) with probability \( \pi \). The additional consumption is obtained by accepting a \( 1/\pi \) unit reduction in his health care in the event that he is ill, which occurs with probability \( \pi \), so the expected cost of this reduction is \( v_2 \). At the optimum these utility changes are exactly offsetting.

The optimal commodity bundles are shown in Figure 2. The locus \( g = 0 \) is upward sloping. Since \( r \) is smaller than 1, the \( f = 0 \) locus lies everywhere above the \( g = 0 \) locus.

The individual and the society have different objectives, and hence make different decisions. The individual faces an uncertain future: he will either be sick or healthy, and he chooses his insurance coverage to make the best of that gamble. The society, on the other hand, provides health care after each person’s health status becomes known. Its health care program considers both the interests of the healthy and the sick, but is designed to ameliorate the differences in their welfare. The redistributive aspect of the two-tier program gives rise to the differences in outcome: both the rich and the poor have greater health care and lower consumption under the two-tier program than under private insurance.

5. One-Tier Public Health Care

One-tier public health care differs from two-tier care in that the rich and the poor receive the same health care and pay the same premiums:

\[ m_R = m_P = m \]  \hspace{1cm} (6)

An optimal one-tier system is a list \((c^1_R, c^1_P, m^1)\) that maximizes \( W(m_R, c_R, m_P, c_P) \) subject to (3), (6) and a set of non-negativity constraints. Again, the restrictions on \( u \) and \( v \) imply that the non-negativity constraints are not binding and can be discarded. The strict concavity of the objective function and the linearity of the constraints implies that a unique interior solution exists. The solution satisfies (3), (6) and the condition

\[ \sum_{i=R,P} v(c_i, m)^{\alpha-1} f(c_i, m) = 0 \]
Figure 3: Optimal one-tier health care is described by the co-ordinates of E and F, while the co-ordinates of A and B describe optimal two-tier health care.

This solution is illustrated in Figure 3. The necessity of treating all agents equally forces the policymaker to compromise. At the optimum, the social value of another unit of health is greater than its cost for the rich, and less than its cost for the poor. Welfare would be higher if $m_R$ were larger and $m_P$ were smaller, but this adjustment is prohibited under one-tier care.

6. A Comparison of One-Tier Care and Private Insurance

The parameters of the model are described by the triplet $(y_P, \pi, \alpha)$. Let the set of admissible triplets is

$$Z \equiv \{(y_P, \pi, \alpha) : y_P \in (0, y/2), \pi \in (0, 1), \alpha \in (0, 1)\}$$

It is also convenient to define a slightly larger set:

$$Z' \equiv \{(y_P, \pi, \alpha) : y_P \in (0, y/2], \pi \in (0, 1], \alpha \in (0, 1]\}$$

Let $W^1(y_P, \pi, \alpha)$ and $W^2(y_P, \pi, \alpha)$ be maximal social welfare under one-tier and two-tier public health care, and let $W^N(y_P, \pi, \alpha)$ be maximal social welfare under private health care insurance. The continuity assumptions on $u$ and $v$ imply that each of these functions is continuous on $Z'$. 

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The maximization problem for one-tier care is obtained by adding a constraint to the maximization problem for two-tier care, and this constraint is generally binding, so

\[ W^2 \geq W^1 \]

with the strong inequality almost always holding. The constraints on private insurance are the same as the constraints on two-tier care, but private insurance maximizes expected utilities rather than social welfare. Consequently,

\[ W^2 \geq W^N \]

with the strong inequality holding in all but a few limiting cases. The limiting cases in which \( W^1 \) or \( W^N \) is equal to \( W^2 \) give some insight into the relative merits of the two systems.

The additional constraint in the one-tier problem, (6), is binding because \( y_P \) is less than \( y_R \), implying that \( W^2 \) is greater than \( W^1 \). Also, \( W^2 \) tends to be greater than \( W^N \) because the strict concavity of the social welfare function implies that there is a benefit to redistribution between the healthy and the sick in each group. Two-tier care differs from private insurance in that it implements this redistribution, and hence tends to be associated with higher levels of social welfare. There are, however, two limiting cases in which these two systems yield the same welfare. If \( \alpha \) is equal to one, so that the social welfare function is linear rather than strictly concave, there is no benefit to redistribution. If \( \pi \) is equal to 1, there is no possibility of redistribution. That is,

\[
W^N(y_P, \pi, 1) = W^2(y_P, \pi, 1) > W^1(y_P, \pi, 1) \quad 0 < y_P < y/2
\]

\[
W^N(y_P, 1, \alpha) = W^2(y_P, 1, \alpha) > W^1(y_P, 1, \alpha) \quad 0 < y_P < y/2
\]

The continuity of these functions on \( Z' \) implies that, so long as there is some income disparity, private insurance leads to higher welfare than one-tier care if either \( \pi \) or \( \alpha \) is sufficiently close to 1. Figure 2 can be used to illustrate this result. As \( \alpha \) rises toward 1, \( r(c_i, m_i) \) rises toward 1 and the \( f = 0 \) locus shifts downwards. This shift causes \( A \) and \( B \) to converge to \( C \) and \( D \), and \( W^2 \) to converge to \( W^N \). Similarly, the \( f = 0 \) and \( g = 0 \) locus converge as \( \pi \) rises toward 1, again causing \( W^2 \) and \( W^N \) to converge.

Now assume that \( \alpha \) and \( \pi \) are smaller than 1. Social welfare can be increased by redistributing from the healthy to the sick, and private insurance fails to exploit this opportunity, so \( W^N \) is less than \( W^2 \). The imposition of the constraint (6)
Figure 4: In the absence of income disparity, one-tier and two-tier care are represented by $E$ while private insurance is represented by $F$.

pushes $W^1$ below $W^2$ whenever there is income disparity, but it does not do so when there is no disparity:

$$W^1(y/2, \pi, a) = W^2(y/2, \pi, a) > W^N(y/2, \pi, a)$$

As before, the continuity of these functions on $Z'$ implies that one-tier health care is preferred to private insurance whenever income disparity is sufficiently small. Figure 4 illustrates the limiting case: the two budget constraints overlap, so that each outcome is represented by a single point. One-tier and two-tier care are both represented by $E$, and hence both systems give rise to the same level of social welfare. Private insurance is associated with a distinctly different outcome, $F$, and yields lower social welfare.

In a related paper [8], I compare health care systems under the twin assumptions that there is no income disparity (or any other ex ante differences among the agents), and that ex post health is a multi-dimensional attribute.\(^5\) I show that under the ex post social welfare criterion, public health care is preferred to private health insurance because it improves the allocation of health care across

\(^5\)Specifically, I assume that health is characterized by a triplet: state of health with treatment, state of health without treatment, and cost of treatment.
the population. The current model gives less dogmatic results: public health care is sometimes preferred to private health insurance, but sometimes it is not.

7. Supplemental Insurance

Some countries, including the United Kingdom and Australia, operate one-tier public health care systems but allow people to buy additional health care insurance on the private market. What are the allocative effects of this system?

A system in which supplemental insurance is offered but no-one buys it is essentially a one-tier system, and a system in which supplemental insurance is offered and everyone buys it has the same properties as a private insurance system. Supplemental insurance only generates a different allocation if the rich choose to purchase it and the poor do not.

The rich will purchase enough insurance to raise their total coverage to \( m_R^N \); the level that they believe to be individually optimal. From the perspective of the rich, the extent of the public health system simply determines the amount of additional coverage that they need. The public health system can therefore be designed to meet the needs of the poor, as these are the only agents who are constrained by it. It follows that a one-tier public health system should offer \( m_P^2 \) units of coverage to everyone, leaving the rich to purchase \( m_R^N - m_P^2 \) units of supplemental coverage. There will be a demand for supplemental insurance if this number is positive, and no demand if it is not. Its sign is determined by the parameters of the model; for example, inspection of Figure 3 shows that \( m_R^N - m_P^2 \) rises as income becomes more unequal or society becomes less averse to inequality.

Now consider welfare. There is no element of \( Z \) for which private insurance is preferred to public health care with supplemental insurance. The two systems generate the same outcome for the rich, but the supplemental insurance system generates a better outcome—indeed, the best outcome—for the poor. Of course, there are elements of \( Z \) for which private insurance is preferred to one-tier care; and for each of these elements, the supplemental insurance system is the best of the three systems.

Likewise, there are elements of \( Z \) for which one-tier care is the best of the three. For any admissible \( y_P \) and \( \pi \), \( m_R^N - m_P^2 \) falls as \( \alpha \) falls from 1. When this difference is arbitrarily close to zero, welfare under the optimal supplemental

\[ \text{The former raises } m_R^N \text{ and reduces } m_P^2, \text{ while the latter (by shifting downward the } f = 0 \text{ locus) reduces } m_P^2 \text{ but leaves } m_R^N \text{ unchanged.} \]
insurance system is arbitrarily close to welfare under a one-tier system in which \( m \) is fixed at \( m^2_P \), and is therefore lower than welfare under the optimal one-tier system. Since supplemental insurance is always preferred to private insurance, one-tier care is preferred to both of them under these circumstances.

There is a latent demand for supplementary insurance under one-tier care if \( m^N_R \) exceeds \( m^1 \). Accommodating that demand always raises social welfare: the rich move closer to \((c^2_R, m^2_R)\), and the public health care program can be changed so that the poor attain \((c^2_P, m^2_P)\). Thus, the existence of a latent demand for supplementary insurance signals that supplementary insurance is the best of the three systems. (However, the absence of a latent demand for supplementary insurance does not imply that one-tier care is the best system.)

These findings are again different from those of the model than emphasizes the allocation of health care across heterogeneous patients [8]. Supplementary insurance is never optimal in that model, as it simply shifts the allocation of health care towards the competitive allocation, and the competitive allocation does not maximize welfare in a society that is averse to inequality. Accommodating a latent demand for supplementary insurance necessarily reduces welfare in that model.

8. Conclusions

A society’s aversion to inequality can be modelled by assuming that the society maximizes a strictly concave ex post social welfare function. If there were no income disparity within the society, public health care would be socially preferred to private health care insurance. The quantity of health care would be greater under the optimal public health care system than under private health care insurance; the additional care would raise welfare by ameliorating the utility gap between the healthy and unhealthy segments of society. However, if income disparity is present, public health care also generates a welfare loss resulting from the frustration of consumer sovereignty. There are then circumstances under which public health care is still socially preferred to private health care insurance, but there are also circumstances under which private health care insurance is preferred to public health care.

A third option is a public health care system under which people have the option to purchase supplementary health care insurance on private markets. This system is always preferred to a private health care insurance system. It is sometimes preferred to a public health care system, but sometimes a public health care system is the best of the three options. A sufficient—but certainly not necessary—
condition for the superiority of the hybrid system is the existence of a latent
demand for supplementary insurance when a public health care system is in place.

References


