Competing Mechanisms: Theory and Applications in Directed Search Markets

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Abstract

This paper studies competing mechanism problems in directed search markets in which multiple principals (e.g., sellers) simultaneously post trading mechanisms to compete for trading opportunities while multiple agents (e.g., buyers) select any particular principal for trading via directed search. Principals can deviate to any arbitrary mechanisms. When a principal’s deviation becomes evident from agents’ messages, the equilibrium mechanisms posted by non-deviating principals allow them to punish a deviating principal with dominant strategy incentive compatible (DIC) direct mechanisms only in a certain class of DIC direct mechanisms. This makes equilibrium analysis tractable and induces rich applications. The theory provides a general characterization of the set of robust equilibrium allocations that are supportable in a game given a class of DIC direct mechanisms that principals can use to punish a deviating principal. Our equilibrium concept is applied to various competing mechanism problems (e.g., competing prices, competing auctions, competing ski-lift pricing, etc.). (JEL C72, D47, D82)
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1 Introduction

Consider a directed search market where multiple principals (e.g., sellers) simultaneously post trading mechanisms to compete for trading opportunities, and multiple agents (e.g., buyers) select any particular principal for trading via directed search. The best known example would be competing auctions where sellers compete over a reserve price (McAfee 1993, Peters and Severinov 1997, Virág 2010, etc.). Buyers participating in auctions are only required to report their willingness to pay, but they also have the market information about competing sellers’ trading mechanisms or their terms of trade. Recently Han (2015) shows that an equilibrium of competing auctions in a large market survives even when a seller can deviate to any arbitrary mechanism. However, by requiring the market information as well as the buyer’s willingness to pay, sellers can actively respond to changes in the market, often to punish the deviator. This makes it possible for sellers to maintain implicit collusion even in a one-shot game without the repeated game effect.

This paper studies competing mechanism problems in directed search markets that are easy to apply and implement. In our model, agents visit all principals as they search for a better deal. In this process, agents can communicate with all principals, but eventually each agent has to select one principal for trading. This paper is interested in an equilibrium where each non-deviator punishes a deviating principal with a dominant-strategy incentive compatible (DIC) direct mechanism only in a class of DIC direct mechanisms, denoted by $\Omega_D$, that are available for punishment in a game. $\Omega_D$ is a subset of all possible DIC direct mechanisms and it can take a variety of forms, depending on the applications; the set of single per-unit prices with/without entry fees or the set of second price auctions with reserve price. We call it an equilibrium with dominant strategy implementable punishment (DSIP).

The DIC property does not depend on the probability distribution of the number of participating buyers so that we can always fix buyers’ truthful reporting to non-deviators without reference to the buyer’s endogenous selection behavior upon a principal’s deviation. Furthermore, if a principal punishes a deviating principal with a DIC direct mechanism, he does not need to know the whole market information that buyers have, only the identity of the deviating principal. For that, a principal can design a deviator-reporting direct mechanism (DDM) where buyers report the identity of the deviating principal and their payoff type, if they indeed select that principal for trading. If the majority of buyers report “no deviation” to principal $\ell$, $\ell$’s DDM assigns an incentive compatible direct mechanism. If the majority of buyers report $j$ as the deviator’s identity to principal $\ell$, $\ell$’s DDM changes his direct mechanism to
a DIC direct mechanism.

For generality, we allow principals to deviate to any arbitrary mechanisms and consider a truthful (incentive consistent) continuation equilibrium upon a principal’s deviation, as in the most of directed search models. The “truthful” part means that buyers report true information to non-deviators and the “incentive consistency” incorporates the lack of coordination among buyers, in the sense that buyers select non-deviators for trading with equal probability if their mechanisms are the same. We are interested in the set of robust equilibrium allocations that can be supported with DSIP even when buyers play the best truthful continuation equilibrium for the deviator. We show that the set of robust equilibrium allocations that are supportable in a game is characterized by the lower bound for each principal j’s payoff. This lower bound is j’s min-max payoff value, where max is taken over j’s all possible Bayesian incentive compatible (BIC) direct mechanisms conditional on other principals’ DIC direct mechanisms, and min is taken over the class of DIC direct mechanisms $\Omega_D$ that other principals can use in order to punish j in the game, based on the best truthful continuation equilibrium upon j’s deviation.

Our theory is sufficiently general to allow principals to implement different incentive compatible direct mechanisms in equilibrium because reporting the identity of a deviating principal makes it possible to impose individualized punishment. However, principals may jointly implement the same direct mechanisms. In this case, following Han (2014), we can show that buyers only need to report whether or not there exists a deviating principal through the binary messages $\{0, 1\}$, but not the identity of the deviating principal. Therefore, if the sum of buyers’ binary messages is greater than the half of the number of buyers, then it triggers DSIP in a DDM.

We apply the notion of a robust equilibrium with DSIP to various competing mechanism problems: competing single prices without capacity constraint, competing auctions (Peters and Severinov 1997, Virág 2010), competing ski-lift pricing with capacity constraint (Peck 2015), and competing mechanisms in online markets (Peters 2015). We are interested in a symmetric equilibrium in which sellers jointly implement the same direct mechanism so that the binary messages $\{0, 1\}$ are enough to convey the relevant market information in DDMs (i.e., whether or not there exists a deviating principal).

Based on our theoretical results, we first show that a (competitive) equilibrium in a game with a restricted set of DIC mechanisms is in fact robust even when sellers can offer any arbitrary mechanisms. For example, with infinitely many buyers and sellers, competing reserve prices given the second price auction format yield a robust equilibrium where sellers’ reserve prices are equal to

\[1\] “0” means no deviation and “1” means deviation by a competing seller.
their cost. Competing in per-unit prices for a divisible good with the capacity constraint (e.g., ski-lift, health club, amusement park ride) yields a robust equilibrium with sufficiently many sellers where the equilibrium per-unit price is the competitive market clearing price.

The key idea in applications is that sellers can design DDMs, that lead to this competitive equilibrium in a restricted set of DIC mechanisms, as the robust punishment if a seller deviates. For example, in a competing auction environment, any reserve prices can be supported in a robust equilibrium with DSIP if they yield the seller’s expected profit no less than the profit associated with setting reserve price to his cost. This is done when each seller offers a DDM that punishes a deviating seller by changing his reserve price to his cost. Given non-deviators’ second price auctions, with reserve price changed to the cost in their DDMs, the deviating seller cannot do any better with offering any arbitrary mechanism than he does by offering the second price auction with reserve price equal to his cost. It is also shown that we can conveniently use the standard technique of the mechanism design for the joint profit-maximization problem. In fact, we can construct a DDM where the reserve price is the monopoly level when no one deviates, but it becomes the competitive level (i.e., the seller’s cost) when a competing seller deviates.

In a competing ski-lift pricing problem, any profile of entry fees along with the competitive per-unit price can be supported in a robust equilibrium with DSIP if it yields the firm’s profit no less than the profit only based on the competitive per-unit price with no entry fees. This is because each firm can offer a DDM such that it allocates its ski-lift according to the competitive per-unit price and a consumer’s entry fee is included in her total payment, but it eliminates entry fees if buyers reveal a competing firm’s deviation. This type of pricing is observed in the form of a firm’s limited time promotion that eliminates entry fees. Firms can charge entry fees as high as the ones that maximize the joint profit given the allocation based on the competitive per-unit price.\(^2\)

A large literature studies equilibrium allocations when sellers are not restricted to offer a certain type of mechanisms. Coles and Eeckhout (2003), for example, consider a model with complete information where each seller has one unit of the object and each buyer with unit demand has the identical and publicly known valuation. They show a continuum of equilibria, including price-posting and auctions with a reserve price. In addition, Virág (2007) shows that

\(^2\)It is convenient to adjust only entry fees but as shown later, we can also use the standard technique of mechanism design for the joint maximization problem for ski-lift. The solution can be supported in a robust equilibrium when each seller’s DDM implements it if no seller deviates but the competitive-per unit price with no entry fee kicks in upon a competing seller’s deviation.
with two possible types of buyers, only ex-post efficient mechanisms such as auctions survive in equilibrium.\footnote{The first application in our paper shows that this is because of the seller’s capacity constraint. A continuum of single prices can be supported in a robust equilibrium with no capacity constraint even when the buyer’s type space is continuous. Without capacity constraint, there is no loss of generality to focus on single prices for the joint profit maximization problem given buyers, each with unit demand.}

Most of the literature on competing mechanisms is interested in general methodologies of constructing mechanisms that allow buyers to transmit their market information along with their types. However, the proposed mechanisms are quite complicated and abstract. For example, principals need to ask agents to describe a deviating seller’s mechanism (Epstein and Peters (1999)) or agents need to recommend the whole mapping of a direct mechanism that a seller needs to choose to punish a deviator (Yamashita (2010)). Alternatively, the players are required to coordinate on complicated messages involving encryption keys (Peters and Troncoso-Valverde (2013)) or they need to rely on the formal language to design contracts that can be contractible directly upon contracts offered by others (Peters and Szentes (2012)).

In an interesting paper, Peters (2015) shows that sellers can use html cookies to exploit the buyers’ market information in a double auction where sellers, each with a homogenous single object, and buyers, each with unit demand, submit their prices and the competing mechanism is built on top of it. Before submitting prices in a double auction, buyers visit all the sellers’ websites in the market. If some seller deviates, buyers revisit non-deviators’ sites to see if they lowered their ask prices as well. Cookies enable a seller to check whether a buyer revisits by simply assigning a binary message from $\{0, 1\}$, where “1” means that a buyer visited before. A seller’s website specifies his ask price as a function of the number of buyers who revisit through an automated program. As the number of revisiting buyers exceeds a certain threshold, non-deviators lower their ask prices to punish the deviator.

Because all of our applications are based on the binary messages for whether or not there exists a deviator, we can implement all the applications in on-line markets without default games such as a double auction. Our applications also show that the competitive equilibrium in a restricted set of DIC mechanisms can serve as the robust punishment in various competing mechanism problems where a deviator cannot gain in any (truthful) continuation equilibrium upon any deviation to any arbitrary mechanism.
2 Preliminaries

There are $J$ principals (e.g., sellers) and $I$ agents (e.g., buyers). Let $\mathcal{J} = \{1, \ldots, J\}$ be the set of principals and $\mathcal{I} = \{1, \ldots, I\}$ the set of agents. Assume that $J \geq 2$ and $I \geq 3$. An agent eventually selects a principal for the principal’s allocation decision. Each agent’s type is independently drawn from a probably distribution $F$ with support $X = [\underline{x}, \bar{x}] \subset \mathbb{R}_+$. Let $u(a, x)$ denote an agent’s payoff when $x \in X$ is her type and $a \in A$ is the allocation decision of the principal who she selects. If an agent does not select principal $j$, he treats the agent’s type as $x^\circ$. Let $\bar{X} = X \cup \{x^\circ\}$. Then, $x \in \bar{X}$ can conveniently characterize the type profile of the agents who select principal $j$. Let $u_j(a, x)$ denote principal $j$’s payoff when $x \in \bar{X}$ is the type profile of the agents who select him and $a \in A$ is his allocation decision.

We allow for a random allocation decision, denoted by $\alpha$. Let $A := \Delta(A)$ be the set of all possible allocation decisions. The nature and scope of a principal’s allocation decision depends on applications. In auction problems, an allocation decision specifies a profile of winning probabilities for bidders and their payments.

2.1 Incentive compatible allocation

We first formulate Bayesian incentive compatibility. Principal $j$’s direct mechanism $\mu_j : \bar{X} \rightarrow A$ specifies his allocation decision conditional on the type messages reported by the agents who select principal $j$. Because agents are ex-ante homogeneous, we assume that mechanisms are anonymous and hence non-discriminatory. Let $\mu = (\mu_1, \ldots, \mu_J)$ be an array of direct mechanisms offered by all principals. Given $\mu$, each agent simultaneously selects one principal and sends him her type message without observing the type messages that other agents send.

Every agent uses the same communication and selection strategies (i.e., symmetry) and chooses principals with equal probability if their direct mechanisms are the same (i.e., non-discrimination over principals’ direct mechanisms). If the agent’s strategy satisfies these two properties, then it is said to be incentive consistent. This assumption captures frictions induced by lack of coordination that is inherent in a decentralized trading process. Most papers for directed search models adopt incentive consistency.

Let $\pi$ denote every agent’s (incentive consistent) selection strategy. In terms of notation, $\pi(\mu_j, \mu_{-j}) : X \rightarrow [0, 1]$ specifies the probability with which an agent selects principal $j$ as a function of her type when principal $j$’s direct mechanism is $\mu_j$ and the distribution of other principals’ mechanisms is $\mu_{-j}$.
The agent’s selection strategy $\pi$ induces the probability distribution over $\bar{X}$ for each principal $j$. Define $z_j(\mu_j, \mu_{-j}))(x)$ as follows

$$z_j(\mu_j, \mu_{-j}))(x) := 1 - \int_x^\bar{x} \pi(\mu_j, \mu_{-j})(s) dF.$$  \hspace{1cm} (1)

The term $z_j(\mu_j, \mu_{-j}))(x)$ is the probability that an agent either has her type below $x$ or selects a principal other than $j$. Then, we can derive the (interim) expected payoff for an agent of type $x$ associated with sending type message $x'$ to principal $j \in J$ upon selecting him as follows:

$$\int_x^\bar{x} \cdots \int_x^\bar{x} \int_A u(a, x) \frac{d\mu_j(x', s_{-1})}{dz_j(\mu_j, \mu_{-j}))(s_2)} \cdots \frac{dz_j(\mu_j, \mu_{-j}))(s_I)}{dz_j(\mu_j, \mu_{-j}))(s_I)},$$

where $s_{-1} = (s_2, \ldots, s_I)$ and each $s_k$ follows the independent probability distribution $z_j(\pi(\mu_j, \mu_{-j})))$.

A profile of direct mechanisms $\mu = (\mu_1, \ldots, \mu_J)$ is BIC (Bayesian incentive compatible) if, for all $j$ and all $x, x' \in X$,

$$\int_x^\bar{x} \cdots \int_x^\bar{x} \int_A u(a, x) \frac{d\mu_j(x', s_{-1})}{dz_j(\mu_j, \mu_{-j}))(s_2)} \cdots \frac{dz_j(\mu_j, \mu_{-j}))(s_I)}{dz_j(\mu_j, \mu_{-j}))(s_I)} \geq \int_x^\bar{x} \cdots \int_x^\bar{x} \int_A u(a, x) \frac{d\mu_j(x', s_{-1})}{dz_j(\mu_j, \mu_{-j}))(s_2)} \cdots \frac{dz_j(\mu_j, \mu_{-j}))(s_I)}{dz_j(\mu_j, \mu_{-j}))(s_I)}.$$

Let $\hat{v}(x, \mu_j, \mu_{-j}, \pi(\mu_j, \mu_{-j}))$ denote the left-hand side on the inequality relation above, which is the agent’s expected payoff associated with sending her true type upon selecting principal $j$.

**Definition 1** A profile of direct mechanisms $\mu = (\mu_1, \ldots, \mu_J)$ and an agent’s selection strategy $\pi$ is an incentive consistent BIC allocation (hereafter simply BIC allocation) if $\mu = (\mu_1, \ldots, \mu_J)$ is BIC given an incentive consistent selection strategy $\pi$ satisfying,

1. $\pi(\mu_j, \mu_{-j}))(x) = 0 \implies \exists \ell \neq j : \hat{v}(x, \mu_\ell, \mu_{-\ell}, \pi(\mu_\ell, \mu_{-\ell})) \geq \hat{v}(x, \mu_j, \mu_{-j}, \pi(\mu_j, \mu_{-j})).$
2. $\pi(\mu_j, \mu_{-j}))(x) > 0 \implies \forall \ell, \hat{v}(x, \mu_j, \mu_{-j}, \pi(\mu_j, \mu_{-j})) \geq \hat{v}(x, \mu_\ell, \mu_{-\ell}, \pi(\mu_\ell, \mu_{-\ell})).$

Let $\Psi_B$ be the set of all BIC allocations.
2.2 Deviator-reporting direct mechanism

It is not easy for non-deviating principals to use BIC direct mechanisms for punishing the deviating principal because Bayesian incentive compatibility is tied to agents’ probability of selecting them, which depends on the deviator’s mechanism. On the other hand, a DIC (dominant strategy incentive compatible) direct mechanism can be easily used in punishing a deviating principal because the DIC property does not depend on agents’ selection strategies: Principal $\ell$’s direct mechanism $\mu_\ell$ is DIC if for all $x, x' \in X$ and all $s_{-1} = (s_2, \ldots, s_I) \in \bar{X}^I$

$$\int_A u(a, x) d\mu_\ell(x, s_{-1}) \geq \int_A u(a, x') d\mu_\ell(x', s_{-1}).$$

(2)

Given this tractability, non-deviating principals may want to punish a deviating principal by implementing DIC direct mechanisms. When principal $j$’s deviation becomes evident, principal $\ell$ ($\ell \neq j$) can always punish principal $j$ with some DIC direct mechanism, say $\mu_j^\ell$. Then, non-deviating principals collectively use the profile of DIC direct mechanisms, $\mu_j^\ell = \{\mu_j^\ell\}_{\ell \neq j}$, to punish principal $j$. We call this type of punishment dominant strategy implementable punishment (DSIP).

Suppose that principals want to support a BIC allocation $({\bar{\mu}}, \pi(\bar{\mu}))$ in equilibrium. For DSIP, each principal $\ell$ does not need to offer a complex mechanism but only a deviator-reporting direct mechanism (DDM) $\bar{\gamma}_\ell$, in which a message space for each agent is $\mathcal{J} \times \bar{X}$. Therefore, each agent reports (i) the identity of a deviating principal (if any) and (ii) either a payoff type or $x^o$. Let $(d_i, x_i)$ denote agent $i$’s report. Note that each agent’s selection decision is included in her type report (i.e., $x^o$ implies that the agent does not select principal $j$). If $d_i = \ell$ is reported to principal $\ell$, it implies that no principal deviated. For any $d = (d_1, \ldots, d_I) \in \mathcal{J}^I$, $\bar{\gamma}_\ell(d, \cdot)$ is a direct mechanism such that

$$\bar{\gamma}_\ell(d, \cdot) := \begin{cases} \mu_j^\ell & \text{if } |\{i : d_i = j \text{ for } j \neq \ell\}| > |\mathcal{J}|/2, \\ \bar{\mu}_\ell & \text{otherwise}. \end{cases}$$

(3)

Given a BIC allocation $({\bar{\mu}}, \pi(\bar{\mu}))$ with $\bar{\mu} = (\bar{\mu}_1, \ldots, \bar{\mu}_J)$, assume that each principal $\ell$ offers a DDM $\bar{\gamma}_\ell$ that satisfies (3). Then, a DDM changes the principal’s direct mechanism contingent on agents’ reports $d$ on the identity of a deviating principal. If more than half of agents report $j$ to principal $\ell$, then principal $\ell$ assigns $\mu_j^\ell$. Otherwise, he assigns $\bar{\mu}_\ell$. Given this rule, it is always optimal for each agent to do the same when every other agent reports the true answer to whether or not there is a deviating principal and which principal deviates if any.

If principals want to support a BIC allocation in equilibrium where principals jointly implement the same incentive compatible direct mechanism, then
principals only need to know whether or not a competing principal has deviated. In this case, a DDM can simply adopt the binary messages \( \{0, 1\} \) (Han 2014) because principals only need to know whether or not there exists a deviating principal, but not the identity of a deviating principal. The message, \( d_i = 0 \), means “no deviation” by competing principals and the message, \( d_i = 1 \), means “deviation by a competing principal.” A non-deviating principal’s DDM simply sums up buyers’ binary messages. If the total number is greater than \( |\mathcal{I}|/2 \), it triggers a punishment as shown in the applications in Section 4.

3 Equilibrium with DSIP

Consider principals’ competition. Each principal \( j \) can post a DDM \( \tilde{\gamma}_j \) such that \( \tilde{\gamma} = (\tilde{\gamma}_1, \ldots, \tilde{\gamma}_J) \) supports a BIC allocation \( (\tilde{\mu}, \pi(\tilde{\mu})) \). Alternatively, he can post a mechanism from a set of arbitrary mechanisms, say \( \Gamma_j \). For mechanisms in \( \Gamma_j \), let \( M \) be a message space that includes all possible messages each agent can send. Let \( H = \{0, 1\} \) be the set of an agent’s selection decisions. If an agent sends \( h = 1 \) to a principal together with any message in \( M \), it implies that the agent selects the principal; sending \( h = 0 \), with any message in \( M \), implies that an agent does not select him. A mechanism is denoted by \( \gamma_j : M^I \times H^I \rightarrow \mathcal{A} \) and specifies principal \( j \)’s allocation decision as a function of all agents’ messages and selection decisions on whether to select principal \( j \). Competition in a directed search market with \( \Gamma = \times_{k=1}^J \Gamma_k \) has the following timing.

1. Each principal \( j \) simultaneously posts either the DDM \( \tilde{\gamma}_j \) or any arbitrary mechanism in \( \Gamma_j \).

2. Each agent searches for all principals’ mechanisms. After that, each agent simultaneously sends a message and her selection decision to every principal with neither observing other agents’ messages nor their selection decisions.

3. According to each principal \( j \)’s mechanism, messages and selection decisions determine his allocation decision.

4. Finally, payoffs are realized.

We adopt the notion of perfect Bayesian equilibrium but strengthen it as shown later in Definition 2. Once all principals post their mechanisms, they define a subgame that agents play. Agents’ communication and selection strategies constitutes a Bayesian Nash equilibrium of a subgame given by each possible
profile of principals’ mechanisms. We call it a continuation equilibrium. A continuation equilibrium induces a principal’s payoff on and off the path following his unilateral deviation.\footnote{It also induces a principal’s payoff off the path following deviations by multiple principals including himself. For the notion of perfect Bayesian equilibrium, it is sufficient to consider unilateral deviation.}

To incorporate lack of coordination among agents, the equilibrium analysis in most of direct search models is based on a truthful incentive consistent continuation equilibrium (hereafter truthful continuation equilibrium) in which (a) every agent uses the same communication and selection strategies\footnote{If a principal’s mechanism is incentive compatible, every agent uses the (same) truthful communication strategy for the principal.} and (b) every agent chooses any non-deviating principals with equal probability if their direct mechanisms are the same (i.e., non-discriminatory over non-deviating principals’ direct mechanisms).

We also utilize a truthful continuation equilibrium. Because a non-deviating principal’s mechanism is a DDM, we can always fix each agent’s truth telling to any non-deviating principal, i.e., her truthful reporting on (i) whether or not there is a deviating principal and, if any, the deviator’s identity, and (ii) her type if she eventually selects the non-deviating principal for trading.\footnote{Truthful type reporting to a non-deviating principal off the path, following a principal’s deviation, is optimal because his DDM assigns a DIC direct mechanism. It is also optimal on the path because DDMs support a BIC allocation \((\bar{\mu}, \pi(\bar{\mu}))\) on the path.}

Fix the profile of DDMs \(\bar{\gamma} = (\bar{\gamma}_1, \ldots, \bar{\gamma}_J)\) that is supposed to support a BIC allocation \((\bar{\mu}, \pi(\bar{\mu}))\) on the equilibrium path. If no principal deviates, agents report no deviation to every principal, and her true payoff type to a principal who she selects according to the selection strategy \(\pi(\bar{\gamma}_j) = \pi(\bar{\mu})\). This is clearly a truthful continuation equilibrium. Then, principal \(j\)’s (ex-ante) expected payoff is

\[
\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j})) := \\
\int_{\mathbb{E}} \cdots \int_{\mathbb{E}} \int_{\mathbb{A}} u_j(a, s) d\bar{\mu}_j(s) dz_j(\pi(\bar{\mu}_j, \bar{\mu}_{-j}))(s_1) \cdots dz_j(\pi(\bar{\mu}_j, \bar{\mu}_{-j}))(s_I),
\]

where \(s = (s_1, \ldots, s_I)\).

Suppose that principal \(j\) unilaterally deviates to offer an arbitrary mechanism \(\gamma_j : M^I \times H^I \rightarrow A\) in \(\Gamma_j\). In a truthful continuation equilibrium, following principal \(j\)’s deviation, let \(c(\gamma_j, \mu_{-j}^I) : X \rightarrow \Delta(M)\) denote every agent’s strategy for communicating with the deviating principal \(j\), when his mechanism is \(\gamma_j\) and non-deviators’ DDMs assign DIC direct mechanisms \(\mu_{-j}^I\) upon \(j\)’s deviation. Therefore, an agent of type \(x\) sends to principal \(j\) a message that is
drawn from a probability distribution \( c(\gamma_j, \mu_{-j}^i)(x) \in \Delta(M) \). Similarly, a mapping \( \pi(\gamma_j, \mu_{-j}^i) : X \to [0, 1] \) is every agent’s selection strategy that describes the probability with which she selects the deviating principal \( j \). An agent of type \( x \) selects principal \( j \) with the probability \( \pi(\gamma_j, \mu_{-j}^i)(x) \in [0, 1] \).

To specify payoffs, it is convenient to utilize a direct mechanism. From principal \( j \)’s mechanism \( \gamma_j \), a communication strategy \( c(\gamma_j, \mu_{-j}^i) \) induces the principal’s direct mechanism \( \beta_c(\gamma_j) : X^I \to A \) as follows. Let \( N \) denote the number of agents who select principal \( j \) (those agents with \( h = 1 \)). Then, for every \( N \leq I \) and every \( x_N = (x_1, \ldots, x_N) \in X^N \), \( \beta_c(\gamma_j) \) is defined as

\[
\beta_c(\gamma_j)(x_N, x^o_N) := \int_{X^{I-N}} \left( \int_{M} \cdots \int_{M} \gamma_j(m, h) dc(\gamma_j, \mu_{-j}^i)(x_N) \times dc(\gamma_j, \mu_{-j}^i)(s_N) \right) d\Phi_{I-N}^{I-N},
\]

where \( x^o_N = (x^o, \ldots, x^o) \) is the array of \( x^o \)'s for \( I - N \) agents who do not select principal \( j \) (i.e., those agents with \( h = 0 \)), and \( s_N = (s_{N+1}, \ldots, s_I) \) is the vector of random variables for those \( I - N \) agents’ types, and \( dc(\gamma_j, \mu_{-j}^i)(x_N) = dc(\gamma_j, \mu_{-j}^i)(x_1) \times \cdots \times dc(\gamma_j, \mu_{-j}^i)(x_N) \), \( dc(\gamma_j, \mu_{-j}^i)(s_N) = dc(\gamma_j, \mu_{-j}^i)(s_{N+1}) \times \cdots \times dc(\gamma_j, \mu_{-j}^i)(s_I) \).

In essence, every agent’s communication strategy \( c(\gamma_j, \mu_{-j}^i) \) converts principal \( j \)’s mechanism \( \gamma_j \) into a direct mechanism \( \beta_c(\gamma_j) \). Given (4), deviating principal \( j \)’s expected payoff associated with deviating to \( \gamma_j \) is

\[
\hat{\Phi}_j(\beta_c(\gamma_j), \mu_{-j}^i, \pi(\gamma_j, \mu_{-j}^i)) := \int_{x} \cdots \int_{x} \int_{A} u_j(a, s) d\beta_c(\gamma_j)(s) dz_j(\pi(\gamma_j, \mu_{-j}^i))(s),
\]

where \( dz_j(\pi(\gamma_j, \mu_{-j}^i))(s) = dz_j(\pi(\gamma_j, \mu_{-j}^i))(s_1) \times \cdots \times dz_j(\pi(\gamma_j, \mu_{-j}^i))(s_I) \) and \( s = (s_1, \ldots, s_I) \).

Now we need to consider how agents communicate with non-deviating principals and how they select them. When principal \( j \) deviates, each agent \( i \) reports \( d_i = j \) to each non-deviating principal \( \ell \ (\ell \neq j) \) and then her true type if she selects him. Let \( \mu^i_{-j, \ell} \) denote the profile of DIC direct mechanisms chosen by principals other than \( \ell \) and \( j \) in their DDMs upon \( j \)’s deviation to \( \gamma_j \). Let \( \pi(\mu^i_{-j, \ell}, \mu^i_{-j, j}) : X \to [0, 1] \) characterize each agent’s strategy for selecting non-deviating principal \( \ell \) when principal \( j \) deviates to \( \gamma_j \). The non-deviators’ DDMs assign \( \mu^i_{-j} = (\mu^i_{-j, j}, \mu^i_{-j, \ell}) \) upon \( j \)’s deviation.

Let \( (c, \pi) \) be a pair of the agent’s strategies for communicating with the deviating principal and for selecting one of the principals upon a principal’s
deviation:
\[
c := \{c(\gamma_j, \mu^j_{-j}) : \forall \gamma_j \in \Gamma_j \},
\]
\[
\pi := \{\pi(\gamma_j, \mu^j_{-j}), \pi(\mu^j_{i,j}, \gamma_j, \mu^j_{-j}) : \forall \gamma_j \in \Gamma_j, \forall \ell \neq j\}.
\]

Let \(O\) be the set of all strategies for communicating with the deviating principal and selecting a principal in a truthful continuation equilibrium off the path, following a principal’s deviation.\(^8\) Recall that \((\bar{\mu}, \pi(\bar{\mu}))\) is a BIC allocation associated with \(\bar{\gamma}\). Now we define a pure-strategy equilibrium \(\{\bar{\gamma}, c, \pi\}\) with DSIP in a market with an arbitrary set of mechanisms \(\Gamma\). Our refinement of perfect Bayesian equilibrium is as follows. When there are multiple truthful continuation equilibria upon a principal’s deviation, agents may play the best continuation equilibrium for the deviating principal. We define a robust equilibrium as follows.

**Definition 2** \(\{\bar{\gamma}, c, \pi\}\) is a robust (pure-strategy) equilibrium with DSIP in a market with an arbitrary set of mechanisms \(\Gamma\) if

1. On the equilibrium path, each agent \(i\) reports “no deviation (i.e., \(d_i = \ell\)” to each principal \(\ell\) and her true type upon selecting each principal \(\ell\) according to \(\pi(\bar{\mu})\), the agent’s selection strategy in a BIC allocation \((\bar{\mu}, \pi(\bar{\mu}))\) that is supported by DDMs \(\bar{\gamma}\).

2. \((c, \pi) \in O\) and, for all \(j\),

\[
\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j})) \geq \sup_{\gamma_j \in \Gamma_j} \left( \sup_{(c', \pi') \in O} \hat{\Phi}_j(\beta_{c'}(\gamma_j), \mu^j_{-j}, \pi'(\gamma_j, \mu^j_{-j})) \right).
\]

(5)

Instead of (5), if we impose that for all \(j\)

\[
\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j})) \geq \sup_{\gamma_j \in \Gamma_j} \hat{\Phi}_j(\beta_{c}(\gamma_j), \mu^j_{-j}, \pi(\gamma_j, \mu^j_{-j}))
\]

(6)

then together with condition 1 in Definition 2 condition (6) defines a (pure-strategy) equilibrium \(\{\bar{\gamma}, c, \pi\}\) with DSIP. (6) simply implies that each principal \(j\)’s equilibrium payoff is no less than his payoff in a given truthful continuation equilibrium \((c, \pi) \in O\) upon his deviation to a mechanism in \(\Gamma_j\). condition (5) is stronger because it requires that each principal \(j\)’s equilibrium payoff be no less than his payoff in all truthful continuation equilibria upon his deviation.

---

\(^{7}\)Note that we fix truth telling to any non-deviating principal given his DDM on and off the path.

\(^{8}\)The “truthful” part means that each agent reports the true identity of the deviator to non-deviating principals and her type upon selecting a deviating principal.
3.1 Pure-strategy equilibrium allocations

Define $\mathcal{M}(\mu^i_{-j})$ as the set of all BIC direct mechanisms that can be induced from all the possible mechanisms offered by the deviating principal $j$ in all truthful continuation equilibria, when non-deviators’ DDMs assign DIC direct mechanisms $\mu^i_{-j}$ after $j$’s deviation:

$$\mathcal{M}(\mu^i_{-j}) := \{\beta_c(\gamma_j) : \forall \gamma_j \in \Gamma_j, \forall c \in \mathcal{O}_c\},$$

where $\mathcal{O}_c$ is the projection of $\mathcal{O}$ onto the space of the agent’s strategies for communicating with the deviating principal. Now define

$$\Pi(\mu_j, \mu^i_{-j}) := \{\pi(\gamma_j, \mu^i_{-j}) : \forall \gamma_j \in \Gamma_j, \forall (c, \pi) \in \mathcal{O} \text{ s.t. } \mu_j = \beta_c(\gamma_j)\}.$$

Therefore, $\Pi(\mu_j, \mu^i_{-j})$ is the set of all strategies of selecting principal $j$ in all possible continuation equilibria, where the agent’s communication strategy induces a BIC direct mechanism $\mu_j$ given $\mu^i_{-j}$ that is assigned by non-deviators’ DDMs upon $j$’s deviation.

The left-hand side of (5) in Definition 2 is the maximum payoff for the deviating principal $j$ across all possible truthful continuation equilibria. It is straightforward to show the following lemma.

**Lemma 1** For all $j$,

$$\sup_{\gamma_j \in \Gamma_j} \left( \sup_{(c', \pi') \in \mathcal{O}} \hat{\Phi}_j(\beta_{c'}(\gamma_j), \mu^i_{-j}, \pi'(\gamma_j, \mu^i_{-j})) \right) = \sup_{\mu_j \in \mathcal{M}(\mu^i_{-j})} \left( \sup_{\pi(\mu_j, \mu^i_{-j}) \in \Pi(\mu_j, \mu^i_{-j})} \hat{\Phi}_j(\mu_j, \mu^i_{-j}, \pi(\mu_j, \mu^i_{-j})) \right).$$

**Proof.** It follows the definition of $\mathcal{M}(\mu^i_{-j})$ and $\Pi(\mu_j, \mu^i_{-j})$. ■

Lemma 1 implies that principal $j$ only needs to consider a deviation to a BIC direct mechanism in $\mathcal{M}(\mu^i_{-j})$ to see if there is a profitable deviation.

Let $\bar{\Omega}_D$ be the set of all DIC direct mechanisms. Suppose that $\Omega_D \subset \bar{\Omega}_D$ is a class of DIC direct mechanisms that non-deviating principals are allowed to use in order to punish a deviating principal in a game. The size of $\Omega_D$ depends on the application we consider and also restrictions imposed in a specific market. For example, it can be as large as the set of all DIC direct mechanisms $\bar{\Omega}_D$, or as small as the set of single prices in some cases. Given $\Omega_D \subset \bar{\Omega}_D$, define the
greatest lower bound for the payoff $\Phi^D_j$ to principal $j$ for the characterization of the robust equilibrium allocations:

$$
\Phi^D_j := \inf_{\mu'_{-j} \in (\Omega_D)^{-1}} \left[ \sup_{\mu_j \in \mathcal{M}(\mu'_{-j})} \left( \sup_{\pi(\mu_j, \mu'_{-j}) \in \Pi(\mu_j, \mu'_{-j})} \hat{\Phi}_j(\mu_j, \mu'_{-j}, \pi(\mu_j, \mu'_{-j})) \right) \right].
$$

(7)

In Theorem 1 we present the characterization of the robust equilibrium allocations supported by DSIP.

**Theorem 1** Given the class of DIC direct mechanisms that principals can use to punish a deviating principal ($\Omega_D \subset \Omega_D$), the set of robust equilibrium allocations that can be supported with DSIP is

$$
\Psi^* := \{(\bar{\mu}, \pi(\bar{\mu})) \in \Psi_B : \hat{\Phi}_j(\bar{\mu}, \bar{\mu}_{-j}, \pi(\bar{\mu}, \bar{\mu}_{-j})) \geq \Phi^D_j \forall j\}.
$$

**Proof.** First of all, an equilibrium allocation $(\bar{\mu}, \pi(\bar{\mu}))$ must be a BIC allocation to be supported in a robust equilibrium and hence $(\bar{\mu}, \pi(\bar{\mu})) \in \Psi_B$. Also each principal $j$’s payoff $\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j}))$ from $(\bar{\mu}, \pi(\bar{\mu}))$ must be no less than $\Phi^D_j$; if $\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j)) < \Phi^D_j$, there is no $\mu'_{-j}$ satisfying (5) in Definition 2 given Lemma 1.

To complete the proof, we need to show that for any given $(\bar{\mu}, \pi(\bar{\mu})) \in \Psi^*$, we can find $\mu'_{-j}$ satisfying (5) in Definition 2. For any $(\bar{\mu}, \pi(\bar{\mu})) \in \Psi^*$, it is clear that there exists $\mu'_{-j}$ such that

$$
\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j})) \geq \sup_{\mu_j \in \mathcal{M}(\mu'_{-j})} \left( \sup_{\pi(\mu_j, \mu'_{-j}) \in \Pi(\mu_j, \mu'_{-j})} \hat{\Phi}_j(\mu_j, \mu'_{-j}, \pi(\mu_j, \mu'_{-j})) \right) \geq \Phi^D_j.
$$

Given Lemma 1, the first inequality above implies that (5) in Definition 2 is satisfied. This completes the proof. ■

Competing auctions or competing prices restrict a seller to offer a mechanism from a set of DIC direct mechanisms (e.g., second price auctions or single prices). Consider a competing mechanism game where each seller is allowed to offer a mechanism only from a class of DIC direct mechanism $\Omega_D \subset \Omega_D$. Applying Lemma 1, we can show the following Corollary:

**Corollary 1** An equilibrium $(\bar{\mu}, \pi)$ in a market with $\Omega_D \subset \Omega_D$ is robust if and only if
\begin{align*}
\hat{\Phi}_j(\bar{\mu}_j, \bar{\mu}_{-j}, \pi(\bar{\mu}_j, \bar{\mu}_{-j})) \geq 
\sup_{\mu_j \in M(\bar{\mu}_{-j})} \left( \sup_{\pi(\mu_j, \bar{\mu}_{-j}) \in \Pi(\mu_j, \bar{\mu}_{-j})} \hat{\Phi}_j(\mu_j, \bar{\mu}_{-j}, \pi(\mu_j, \bar{\mu}_{-j})) \right).
\end{align*}

\textbf{Proof.} A DIC direct mechanism \( \bar{\mu}_\ell \) for each \( \ell \) can be thought of as a DDM that always assigns \( \bar{\mu}_\ell \) whether or not a competing seller deviates. Replacing \( \mu_j - j \) with \( \bar{\mu}_{-j} \) in Lemma 1, we can apply it to establish (8). \qed

Corollary 1 shows that the seller \( j \) only needs to consider a BIC direct mechanism to see if there is any profitable deviation, given DIC direct mechanisms offered by the other sellers’ DIC direct mechanisms \( \bar{\mu}_{-j} \).

### 3.2 Mixed-strategy equilibrium allocations

So far we have studied an equilibrium with DSIP where principals employ pure strategies. The results can be extended for a mixed-strategy equilibrium with DSIP.

Consider a situation where each principal \( \ell \) employs a mixed strategy \( \delta_\ell \) for posting a DDM such that the support of \( \delta_\ell \) includes only DDMs that always assign the same DIC direct mechanism \( \mu_\ell \) to punish principal \( j \)'s deviation, but different DIC direct mechanisms when no competing principals deviate. For example, suppose that principal \( \ell \) offers DDMs \( \bar{\gamma}_\ell \) and \( \bar{\gamma}_\ell' \) with equal probability. Then, \( \bar{\gamma}_\ell \) assigns a DIC direct mechanism \( \mu_\ell \), but \( \bar{\gamma}_\ell' \) assigns a different DIC direct mechanism \( \mu_\ell' \) on the equilibrium path. However, they all assign the same DIC direct mechanism \( \mu_\ell' \in \Omega_D \) when more than half of agents report principal \( j \)'s deviation.

If no one deviates, principal \( j \)'s payoff becomes
\[
\Phi_j(\delta_j, \delta_{-j}, \pi) := \int_{\Omega_D} \int_{(\Omega_D)^{-1}} \hat{\Phi}_j(\mu_j, \mu_{-j}, \pi(\mu_j, \mu_{-j})) d\delta_{-j} d\delta_j.
\]
If principal \( j \) deviates to \( \gamma_j \), all DDMs in the support of principal \( \ell \)'s mixed strategy \( \delta_\ell \) assigns \( \mu_\ell' \in \Omega_D \). Therefore, principal \( j \)'s payoff becomes
\[
\hat{\Phi}_j(\beta_\ell(\gamma_j), \mu_{-j}^2, \pi(\gamma_j, \mu_{-j}^2)).
\]
This leads to the following definition of a robust mixed-strategy equilibrium.

\textbf{Definition 3} \( \{\delta, c, \pi\} \) is a robust mixed-strategy equilibrium with DSIP in a market with an arbitrary set of mechanisms \( \Gamma \) if,
1. On the equilibrium path, each agent $i$ reports “no deviation (i.e., $d_i = \ell$)” to each principal $\ell$ and her true type upon selecting each principal $\ell$ according to $\pi$.

2. For all $j$,

$$\Phi_j(\delta_j, \delta_{-j}, \pi) \geq \sup_{\gamma_j \in \Gamma_j} \sup_{(\gamma', \mu') \in \Omega} \Phi_j(\beta(\gamma_j), \mu^\ell_{-j}, \pi'(\gamma_j, \mu_{-j})).$$

Since we consider a robust mixed-strategy equilibrium with DSIP, where principals assign DIC direct mechanisms on the path, we can provide the characterization of random DIC allocations that can be supported in a robust mixed-strategy equilibrium with DSIP.

**Theorem 2** Given the class of DIC direct mechanisms that principals can use to punish a deviating principal ($\Omega_D \subset \Omega_D$), any random DIC allocation in

$$\Psi^*_D := \{ \delta, \pi \in \Psi_D : \Phi_j(\delta_j, \delta_{-j}, \pi) \geq \Phi^D_j \forall j \},$$

can be supported in a robust mixed strategy equilibrium allocation with DSIP.

In order to see if there is a profitable deviation, principal $j$ needs to consider a deviation to a BIC direct mechanism in $\mathcal{M}(\mu^\ell_{-j})$, where $\mu^\ell_{-j}$ is the DIC direct mechanisms that non-deviators use to punish him. If the BIC-DIC equivalence holds in terms of expected payoffs, then $\mathcal{M}(\mu^\ell_{-j})$ can be replaced with the set of all DIC direct mechanisms for any given $\mu^\ell_{-j}$ throughout the paper.\(^9\)

### 4 Applications

We study how to apply the notion of an equilibrium with DSIP in various problems. Consider markets where sellers sell homogeneous products. We are interested in a symmetric equilibrium where sellers jointly implement the same direct mechanism in equilibrium. In this case, the relevant market information is whether there exists a deviation by a competing seller. Han (2014) adopts the binary message, i.e., 0 or 1, for buyers to convey this relevant market information. We also follow this approach in constructing DDMs in this section. Specifically, 0 means no deviation and 1 means deviation by at least one competing principal.

\(^9\)The BIC-DIC equivalence for the single principal and multiple agents (Gershkov et al. (2015)) can be extended to the case with multiple principals and agents. The result is available upon request.
The key intuition in applications is that the competitive equilibrium in a restricted set of DIC mechanism is robust in various problems so that a principal cannot gain in any truthful continuation equilibrium upon deviation to any arbitrary mechanism. This implies that sellers or firms can design DDMs that assign the DIC mechanism associated with that competitive equilibrium as the punishment when buyers reveal a competing seller’s deviation. Then, the best thing that a deviator can do is to offer what he could have offered in that competitive equilibrium in a restricted set of DIC mechanism even when he can offer any arbitrary mechanism. Therefore, any terms of trade can be jointly implemented by the sellers or firms if they generate the profit no less than the profit generated in the competitive equilibrium with a restricted set of DIC mechanisms. Further, we show that the standard technique of the mechanism design can be used for the joint profit maximization problem and also identify simple ways to implement various terms of trades in applications. Finally, we show how our applications can be implemented in on-line markets.

4.1 Competing prices

$J$ sellers can produce homogeneous products at a constant marginal cost, normalized to zero. Each seller’s reservation profit is zero. Assume that $kJ$ buyers are looking for the product. Each buyer has the unit demand for the product. If she consumes the product and pays $p$, her utility is $x - p$, where $x$ is the buyer’s valuation that follows a probability distribution $F$ over $[0, 1]$.

Suppose that, once a buyer selects him, a seller makes the probability $q(x)$ that a buyer acquires the product and her payment $p(x)$, as a function of only her reported valuation. Then, $\{q(\cdot), p(\cdot)\}$ is a DIC direct mechanism. Consider the sellers’ joint profit maximization problem where each seller relies on the same DIC direct mechanism $\{q(\cdot), p(\cdot)\}$ to sell his product to each buyer. Then, each buyer selects each seller with equal probability and hence each seller’s expected profit is

$$\int_0^1 \left[ \left( x - \frac{1 - F(x)}{f(x)} \right) q(x) \right] f(x) dx.$$

Therefore, the joint profit-maximization problem becomes the optimal selling problem where each seller acts as a monopolist with an equal market share $k$. If the monotone hazard rate, $h(x) = \frac{f(x)}{1 - F(x)}$ is increasing in $x$, then we have the “bang-bang” solution (see Fudenberg and Tirole (1991)); $q(x) = 1$ if $x \geq x^* > 0$.

10Because buyers’ utility functions are quasilinear in money and each buyer’s valuation follows an identical and independent probability distribution $F$, there is no loss of generality to focus on this class of mechanisms $\{q(\cdot), p(\cdot)\}$ for the joint profit maximization problem, even if principals can use any arbitrary mechanisms.
and \( q(x) = 0 \) otherwise, where \( x^* = \max \left\{ x : x - \frac{1-F(x)}{f(x)} = 0 \right\} \). Subsequently, the optimal selling mechanism is the take-it-or-leave-it single price offer \( x^* \).

If the sellers are restricted to post only single prices, \( x^* \) cannot be sustained as the equilibrium price because each seller has an incentive to undercut his price slightly. Suppose that each seller now offers a DDM that assigns \( p' \) if the majority of buyers report “deviation by a competing seller, (i.e., \( d_i = 1 \))”, that is, \( \sum_{i=1}^{kJ} d_i \geq kJ/2 \); otherwise, it assigns \( x^* \). We need to choose \( p' \).

Suppose that \( p' \) is the competitive price, i.e., zero. For the robustness property, it is important to note that a deviating seller cannot earn a positive profit in any truthful continuation equilibrium upon deviating to any arbitrary mechanism when non-deviating sellers’ prices are \( p' = 0 \). Therefore, the monopoly price \( x^* \) can be sustained as an equilibrium price in a robust equilibrium with DSIP where the DDM lowers the price to the competitive level once the majority of buyers report, i.e., “\( d_i = 1 \)”. Indeed, any price \( p \in [0, x^*] \) can be sustained as an equilibrium price in a robust equilibrium where sellers use the DDMs.

### 4.2 Competing auctions

In the previous example, each seller has no capacity constraint. What if each seller can produce at most a single unit? Let us continue to assume that each seller’s cost is normalized to zero and that each buyer’s valuation follows a probability distribution \( F \) over \([0, 1]\).\(^{11}\)

The literature has studied competing auctions where sellers are restricted to choose their reserve prices in the second price auction. This has created challenges in several aspects of modelling. The first challenge is that the existence of a pure-strategy equilibrium of competing auctions is not generally guaranteed in a market with a finite number of sellers and buyers. This is because a seller’s best-response reserve price function, given the others’ auction offers, is often discontinuous (Burguet and Sákovics 1999). Therefore, the literature has focused on competing auctions in a large market with infinitely many sellers and buyers (Peters and Severinov 1997, Peters 1997, Virag 2010). In such a large market, a single seller’s mechanism has no impact on the market payoff that a buyer can expect. It has been shown that there exists a pure strategy equilibrium where each seller sets up her reserve price equal to her valuation of the object. However, the robustness of the pure-strategy equilibrium still needed to be determined. This is the second challenge.\(^ {12}\)

\(^{11}\)For simplicity, we set \([0, 1]\) as the support of the buyer’s valuation and an identical cost for sellers. However, the buyer’s valuation can belong to any interval \([a, b]\) and the seller’s cost to any interval \([c, d]\) as long as \([c, d]\) are contained in \([a, b]\). (See Peters 1997).

\(^{12}\)Virág (2007) studies these two problems with two sellers and two buyers. He shows that
For any given reserve price offer, a truthful continuation equilibrium is well defined (Peters and Severinov 1997, Peters 1997, Virag 2010). Let $\mu_\ell(r)$ denote seller $\ell'$s second price auction with reserve price $r$ for all $\ell$. Given $J$ sellers in the market, $\hat{\Phi}_j^J(\mu_j(r), \mu_{-j}(r), \pi(\mu_j(r), \mu_{-j}(r)))$ denotes seller $j$'s expected profit if every seller sets up his reserve price equal to $r$. In this case, any buyer whose valuation above $r$ selects each seller with equal probability, and the probability that a buyer with valuation $x$ wins the object from a seller is

$$\left[1 - \frac{1 - F(x)}{J}\right]^{kJ-1}.$$  

As $J \to \infty$, this winning probability approaches $e^{-k(1-F(x))}$. According to Peters and Severinov (p.166, 1997), the limit payoff becomes

$$\lim_{J \to \infty} \hat{\Phi}_j^J(\mu_j(0), \mu_{-j}(0), \pi(\mu_j(0), \mu_{-j}(0))) = k \int_r^1 \left[ \left( x - \frac{1 - F(x)}{f(x)} \right) e^{-k(1-F(x))} \right] f(x)dx.$$  

The pure-strategy equilibrium of competing auctions in a large market exists and is competitive in the sense that the equilibrium reserve price is equal to the seller’s cost, i.e., $r = 0$. That is,

$$\lim_{J \to \infty} \hat{\Phi}_j^J(\mu_j(0), \mu_{-j}(0), \pi(\mu_j(0), \mu_{-j}(0))) \geq \lim_{J \to \infty} \sup_{r \in [0,1]} \hat{\Phi}_j^J(\mu_j(r), \mu_{-j}(0), \pi(\mu_j(r), \mu_{-j}(0))).$$

Corollary 1 in Section 3 provides a way to examine the robustness of the (pure-strategy) equilibrium of competing auctions. According to it, the equilibrium is robust if

$$\lim_{J \to \infty} \hat{\Phi}_j^J(\mu_j(0), \mu_{-j}(0), \pi(\mu_j(0), \mu_{-j}(0))) \geq \lim_{J \to \infty} \sup_{\mu_j \in M_j(\mu_{-j}(0))} \left( \sup_{\pi(\mu_{-j}(0)) \in \Pi(\mu_j, \mu_{-j}(0))} \hat{\Phi}_j^J(\mu_j, \mu_{-j}(0), \pi(\mu_j, \mu_{-j}(0))) \right).$$  \hspace{1cm} (9)

Han (2015) shows that (9) is embedded in the notion of the (pure-strategy) equilibrium of competing auctions in a large market (Peters 1997), so that the equilibrium in a large market is robust.
By using DDMs, we can show that more allocations can be supported. For notational simplicity, let $\Phi^r$ be the seller’s expected profit in the truthful continuation equilibrium where every seller’s reserve price is $r$, i.e.,

$$\Phi^r := k \int_0^1 \left[ \left( x - \frac{1 - F(x)}{f(x)} \right) e^{-k(1-F(x))} \right] f(x) dx.$$

**Proposition 1** Any $(\mu(r), \pi(\mu(r)))$ can be supported as a robust equilibrium with DSIP if

$$r \in R := \{ r \in [0, 1] : \Phi^r \geq \Phi^0 \}.$$

**Proof.** Consider any $r \in R$. Construct a DDM where the seller’s reserve price is 0 when the majority of buyers report “deviation by a competing seller (i.e., $d_i = 1$)”; otherwise, it is equal to $r$. Suppose that every seller offers such a DDM. If no one deviates, each seller’s expected profit is $\Phi^r$. If a seller deviates, non-deviators’ reserve prices all become zero. Given second price auctions with zero reserve price offered by the other sellers, $\Phi^0 = \lim_{J \to \infty} \hat{\Phi}_J^f(\mu_j(0), \mu_{-j}(0), \pi(\mu_j(0), \mu_{-j}(0)))$ is the highest possible expected profit that a deviator can expect amongst all truthful continuation equilibria upon his deviation to any arbitrary mechanism because of (9). Therefore, $(\mu(r), \pi(\mu(r)))$ with $r \in R$ is supported as a robust equilibrium with DSIP. ■

Clearly, $R$ is non-empty because 0 is always in $R$. However, in general, $R$ will have more reserve prices that can be supported in an equilibrium with DSIP. Let $r^*$ be the jointly profit maximizing reserve price, that is,

$$r^* := \arg \max_{r \in [0,1]} \int_0^1 \left[ \left( x - \frac{1 - F(x)}{f(x)} \right) e^{-k(1-F(x))} \right] f(x) dx.$$

$(\mu(r^*), \pi(r^*))$ can be supported by using DDMs where the seller’s reserve price is 0 when the majority of buyers report “deviation by a competing seller (i.e., $d_i = 1$)”; otherwise, it is equal to $r^*$. If the monotone hazard rate $\frac{f(x)}{1-F(x)}$ is increasing, then $r^*$ is equal to $\max \left\{ r \in [0, 1] : x - \frac{1 - F(x)}{f(x)} = 0 \right\}$. In this case, $R = [0, r^*]$. Buyers’ utility functions are quasilinear in money and their valuation follows the identical and independent probability distribution. Hence, following the standard technique of mechanism design, the second price auction with reserve price $r^*$ is the optimal selling mechanism for the joint profit maximization problem even if sellers can use any arbitrary mechanisms.
4.3 Competing ski-lift pricing

Peck (2015) studies oligopolistic competition between firms with the fixed production capacity and a continuum of consumers, each of whom demands multiple units and can select one firm during the market period. Examples include competition among firms who own mountains for downhill skiing with a chair lift that can accommodate a fixed number of ski runs during a day, firms that sell amusement park rides or, to some extent, health club classes. In contrast to the classical paper by Barro and Romer (1987), Peck considers a competing mechanism game among firms who non-cooperatively post incentive compatible direct mechanisms.

In his model, \( n \) firms have the same capacity, normalized to one and no costs. There are \( I \) types of consumers and a continuum of consumers of each type. The measure of type \( i \) consumers are denoted by \( k_i n \). Each consumer of type \( i \) has a quasilinear utility function \( u_i(x_i) - P_i \), where \( x_i \) is the consumption of the (divisible) good and \( P_i \) is the payment to the firm. \( u_i(x_i) \) satisfies \( u_i'(x_i) > 0 \) and \( u_i''(x_i) < 0 \) for all \( x_i \). Assume that each consumer’s reservation utility is normalized to zero. First, Peck (2015) considers the Price-Per-Unit (PPU) game, where each firm is restricted to post a single per-unit price. If a consumer of type \( i \) faces price \( p \), she will choose the quantity of the good that satisfies

\[
    u_i'(x_i) = p
\]

and let its solution be denoted by \( d_i(p) \). In terms of willingness to pay, it is assumed in Peck (2015) that \( i > h \) which implies that \( d_i(p) > d_h(p) \) for all \( p \).

Given \( p^j \), per-unit price posted by firm \( j \), let \( \pi_i^j \) denote the probability that each consumer of type \( i \) chooses firm \( j \). If there is no excess demand, i.e., \( \sum_{i=1}^{I} k_i n \pi_i^j d_i(p^j) \leq 1 \), then consumption is given by \( x_i = d_i(p^j) \). If there is excess demand, i.e., \( \sum_{i=1}^{I} k_i n \pi_i^j d_i(p^j) > 1 \), then some consumers are rationed in a way that their consumption is given by \( x_i = \min[d_i(p^j), \bar{x}^j] \), where \( \bar{x}^j \) is the maximum quantity that any consumer can choose and it is the unique solution to

\[
    \sum_{i=1}^{I} k_i n \pi_i^j \min[d_i(p^j), \bar{x}^j] = 1. \tag{10}
\]

The competitive price, denoted by \( p^c \), is the unique solution to \( \sum_{i=1}^{I} k_i n d_i(p^c) = n \). Proposition 1 in Peck (2015) shows that for sufficiently large \( n \), the PPU game has the competitive equilibrium in which all firms choose \( p^c \) and consumers choose each firm with equal probability on the equilibrium path. We can establish the robustness of the competitive equilibrium as follows.
Proposition 2  For sufficiently large \(n\), the competitive equilibrium price \(p^c\) can be supported as a robust equilibrium with DSIP.

Proof. The fixed competitive price \(p^c\) can be thought of as a DIC direct mechanism because the price does not depend on consumers’ type messages. Let \(\bar{\mu}^c\) be the DIC direct mechanism that offers the competitive price \(p^c\) regardless of participating consumers’ type reports. According to Corollary 1 in Section 3, we need to show that

\[
\hat{\Phi}_j(\bar{\mu}^c_j, \bar{\mu}^c_{-j}, \pi(\bar{\mu}^c_j, \bar{\mu}^c_{-j})) \geq \sup_{\mu_j \in \mathcal{M}(\bar{\mu}^c_{-j})} \left( \sup_{\pi(\mu_j, \bar{\mu}^c_{-j}) \in \Pi(\mu_j, \bar{\mu}^c_{-j})} \hat{\Phi}_j(\mu_j, \bar{\mu}^c_{-j}, \pi(\mu_j, \bar{\mu}^c_{-j})) \right). \tag{11}
\]

to prove the robustness. Lemma 1 in Peck (2015) is used to show that there exists a (truthful) continuation equilibrium upon firm \(j\)’s deviation to \(\mathcal{M}(\bar{\mu}^c_{-j})\), given the fixed competitive price offered by other firms. Proposition 3 in Peck (2015) proves (11) showing that firm \(j\) cannot gain in any given continuation equilibrium upon firm \(j\)’s deviation to \(\mathcal{M}(\bar{\mu}^c_{-j})\).

The intuition behind the proof of Proposition 3 in Peck (2015) is as follows. He shows that for a deviating firm, it is always better to attract only type-1 consumers because their willingness to pay is higher. Therefore, the upper bound of the maximum payoff upon deviation, i.e., the right-hand side of (11), is characterized by the payment from type-1 consumers and the probability \(\pi_j^1\) that maximize firm \(j\)’s profit in a continuation equilibrium upon his deviation, given the constraint that type-1 consumers are indifferent between buying the good from the deviator and being rationed at any non-deviating firm.

If the deviating firm raises its price for type-1 consumers slightly above the competitive price, more consumers visit non-deviating firms. This makes type-1 consumers visiting non-deviating firms rationed, whereas type-1 consumers visiting the deviating firm can purchase as many units as they want at the higher price. Peck shows that if \(n\) is large, the increase in the deviator’s profit due to the price increase is not large enough to compensate the decrease in the quantity sold. Therefore, Peck (2015) proves the inequality in (11). We can then apply Corollary 1 in Section 3 to complete the proof of Proposition 2.

Peck identified another robust equilibrium, under some additional conditions on the consumer’s valuation function \(u_i(\cdot)\), in which firms offers a modified market clearing (MMC) mechanism with entry fees. Let \(\rho^j_i\) be the measure of consumers who select firm \(j\) with \(\rho^j = \sum_{i=1}^I \rho^j_i\). A market clearing mechanism with entry fee is a set of entry fees \(E^j_i\) satisfying: (i) \(x^j_i(\rho^j_i) = d_i(p(\rho^j))\), (ii)
$P^j_i(\rho^i) = p(\rho^j)d_i(p(\rho^i)) + E^j_i$ and (iii) $p(\rho^i)$ is determined by the market clearing condition, $\sum_{i=1}^I p^j_i d_i(p(\rho^i)) = 1$. Therefore, given the measures of consumers who select firm $j$, consumer $i$ consumes $x^j_i(\rho^j_i)$ by paying $P^j_i(\rho^j_i)$. If $E^j_i$ varies across types, the mechanism might be not incentive compatible when the measure of arriving consumers is small enough that the market clearing price is near zero. A MMC mechanism with entry fees modifies a market clearing mechanism in a way that the entry fees fully apply in an $\epsilon$ neighborhood of $p^c$ and linearly drop to zero as the price reaches $p^c - 2\epsilon$ or $p^c + 2\epsilon$.

The equilibrium MMC mechanism fixes the entry fee as

$$E^*_i = -\frac{d_i(p^c)}{(n-1) \sum_{h=1}^I k_h \frac{\partial d_h(p^c)}{\partial p}}.$$

Because each firm offers the same MMC mechanism with entry fees, consumers select firms with equal probability so that the equilibrium per-unit price becomes the competitive price $p^c$ and the profile of equilibrium entry fees is $E^* = (E^*_1, \ldots, E^*_I)$.

We can actually show that firms can jointly implement many more profiles of entry fees given the competitive per-unit price $p^c$. Consider a profile of entry fees $(E_1, \ldots, E_I)$, along with the quantities based on the competitive prices, and the rationing rule in (10) with $p^j = p^c$. Let $x^c_i := d_i(p^c)$ be the quantity of the good for the consumer of type $i$ and $P_i = p^c x^c_i + E_i$ be the payment by the consumer of type $i$. Then, let $\{(x^c_1, P_1), \ldots, (x^c_I, P_I)\}$ denote a market clearing menu of quantity and payment pairs that each consumer can choose.\(^\text{13}\) If all firms sell their goods according to the same market clearing menu, each firm’s profit is $\sum_{i=1}^I k_i (p^c x^c_i + E_i)$. Let $\mathcal{E}$ be the set of all profiles of entry fees that satisfy

$$\mathcal{E} := \left\{(E_1, \ldots, E_I) : \begin{array}{l}
(i) u_i(x^c_i) - P_i \geq 0 \text{ for all } i \\
(ii) u_i(x^c_i) - P_i \geq u_i(x^c_h) - P_h \text{ for all } i, h
\end{array}\right\}.$$

The first constraint is the individual rationality condition and the second constraint is the incentive compatibility.

\textbf{Proposition 3 Any profile of entry fees in} $\mathcal{E}$ can be supported along with the competitive price $p^c$ in a robust equilibrium with DSIP.

\(^{13}\)Even if the rationing rule \((10)\) is imposed for a market clearing menu it is never used in equilibrium.
Proof. In order to support a profile of entry fees \((E_1, \ldots, E_I)\), it must be in \(\mathcal{E}\) because it must satisfy the individual rationality and the incentive compatibility. Pick any profile of entry fees \((E_1, \ldots, E_I) \in \mathcal{E}\). Construct a DDM mechanism where the firm offers a single competitive price \(p^c\) without entry fees and the rationing rule based on (10), with \(p^j = p^c\) if the majority of buyers report “deviation by a competing firm (i.e., “1”); otherwise it offers \:\{(x^c_1, P_1), \ldots, (x^c_I, P_I)\}\ with \(P_i = p^c_i x^c_i + E_i\) and the rationing rule based on (10) with \(p^j = p^c\). If no one deviates, consumers select firms with equal probability and each firm’s profit becomes \(\sum_{i=1}^I k_i (p^c_i x^c_i + E_i)\). If some firm deviates, non-deviators offers the single competitive price \(p^c\). Then, \(\sum_{i=1}^I k_i p^c_i x^c_i\) is the maximum profit that the deviator can receive in any continuation equilibrium upon his deviation according to Proposition 2. Therefore, any profile of entry fees \((E_1, \ldots, E_I) \in \mathcal{E}\) can be supported as a robust equilibrium with DSIP if and only \(\sum_{i=1}^I k_i E_i \geq 0\). 

We can ask, given the competitive per-unit price \(p^c\), what are the optimal entry fees that jointly maximize each firm’s profit,

\[
\sum_{i=1}^I k_i P_i = \sum_{i=1}^I k_i p^c_i x^c_i + \sum_{i=1}^I k_i E_i,
\]

among those in \(\bar{\mathcal{E}}\)? Given the single crossing property of \(u_i(\cdot)\), the problem is standard: with the optimal entry fees, every local downward incentive compatibility is binding and the individual rationality condition for the consumer of type \(I\) is binding: \(u_i(x^c_i) - P_i \geq u_i(x^c_{i+1}) - P_{i+1}\) for all \(i\) and \(u_I(x^c_I) - P_I = 0\). From those two conditions, the optimal entry fees \(\bar{E} = (\bar{E}_1, \ldots, \bar{E}_I)\) are

\[
\bar{E}_I = u_I(x^c_I) - p^c x^c_I,
\]

\[
\bar{E}_i = -p^c x^c_i + \sum_{g=0}^{i-1} \left[ u_i + g(x^c_{i+g}) - u_i + g(x^c_{i+g+1}) \right] + p^c x^c_i + \bar{E}_i \text{ for all } i \neq I.
\]

As shown in the proof of Proposition 3 it is very easy to implement entry fees in \(\bar{\mathcal{E}}\). For example, the optimal entry fee \((\bar{E}_1, \ldots, \bar{E}_I)\) with the competitive per-unit price \(p^c\) is implemented in the following way: If buyers reveal that no competing firm has deviated, firms charge \((\bar{E}_1, \ldots, \bar{E}_I)\) along with the competitive per-unit price \(p^c\) through the menu \([(x^c_1, \bar{P}_1), \ldots, (x^c_I, \bar{P}_I)]\) with the rationing rule. If buyers reveal that a competing firm has deviated, firms eliminate the entry fees and supply their capacity solely based on the competitive per-unit price \(p^c\) with the rationing rule. This type of changes in pricing is observed in practice. For example, sometimes a firm offers a limited time promotion that eliminates entry fees.
Adjusting only entry fees is quite practical. However, we can identify more allocations that can be supported in a robust equilibrium with DSIP. If all firms’ menus are the same, then consumers select firms with equal probability so that $k_i$ is the measure of consumers of type $i$ who select a firm. For the joint implementation of terms of trade, this implies that even when arbitrary mechanisms are available, we can focus on menus of quantity and payments $[(x_1, P_1), \ldots, (x_I, P_I)]$ that satisfy (i) $\sum_{i=1}^{I} k_i x_i \leq 1, x_i \geq 0$, (ii) $u_i(x_i) - P_i \geq 0$ for all $i$, (iii) $u_i(x_i) - P_i \geq u_i(x_h) - P_h$ for all $i, h$. Let $\mathcal{C}$ be the set of all possible menus satisfying (i)-(iii).

**Proposition 4** Any menu in

$$\bar{\mathcal{C}} = \left\{ [(x_1, P_1), \ldots, (x_I, P_I)] \in \mathcal{C} : \sum_{i=1}^{I} k_i P_i \geq \sum_{i=1}^{I} k_i p^c d_i(p^c) \right\}$$

can be supported in a robust equilibrium with DSIP.

**Proof.** Choose any menu $[(x_1, P_1), \ldots, (x_I, P_I)] \in \bar{\mathcal{C}}$. Construct the corresponding DDM that assigns the single competitive per-unit price $p^c$ without entry fees based on the rationing rule (10), with $p^f = p^c$ if the majority of buyers report “deviation by a competing firm, (i.e., “1”); otherwise $[(x_1, P_1), \ldots, (x_I, P_I)]$. If no one deviates, consumers select firms with equal probability and the type-$i$ consumer choose $(x_i, P_i)$ given conditions (ii) and (iii). If a firm deviates, non-deviators’ DDMs assign the competitive per-unit price $p^c$ without entry fees. Given the competitive per-unit price offered by non-deviators, the deviating firm cannot do any better with offering any arbitrary mechanism than it does with offering the same competitive per-unit price, according to Proposition 2. Therefore, $\sum_{i=1}^{I} k_i p^c d_i(p^c)$ is the maximum payoff that the deviating firm can receive upon deviation to any arbitrary mechanism. Because the expected profit $\sum_{i=1}^{I} k_i P_i$ from the menu in $\in \bar{\mathcal{C}}$ is no less than $\sum_{i=1}^{I} k_i p^c d_i(p^c)$, no firm has an incentive to deviate from his DDM. ■

One can find the jointly profit maximizing menu by solving the problem

$$\max_{[(x_1, P_1), \ldots, (x_I, P_I)] \in \mathcal{C}} \sum_{i=1}^{I} k_i P_i,$$

and construct the corresponding DDM to implement it in a robust equilibrium with DSIP.\textsuperscript{15} Clearly, $\mathcal{E} \subset \bar{\mathcal{C}}$ so that there are extra menus that can be supported

\textsuperscript{14}For the menu $[(x_1, P_1), \ldots, (x_I, P_I)]$ that is implemented in equilibrium, we do not need a rationing rule. However, one can impose a rationing rule such as (10).

\textsuperscript{15}Because buyers’ utility functions are quasilinear in money and there is a continuum of consumers for each type, there is no loss of generality to focus on menus in $\mathcal{C}$ for the joint profit maximization problem.
in \( \tilde{C} \) but those extra menus may requires quantities for consumers that are not the same as one that are demanded by the consumers given the competitive price. However, any menus in \( \tilde{E} \) are based on quantities demanded given the competitive unit-price. Because the punishment in the DDM is based on the competitive per-unit price (without entry fees), it is easier to implement the menus in \( \tilde{E} \) by eliminating only entry fees if there is a deviation by a competing firm.

4.4 Competition in on-line markets

On-line sellers can keep track of buyers’ search history based on html cookies, which are used in many different ways. For example, websites may share the information collected from cookies to increase profits (Ghosh et al. 2012). When the value of the product is not perfectly known to all the buyers, sellers’ websites exploit cookies in determining how much information of the product should be revealed to buyers because search history in cookies reflects how well informed a buyer is about the products in the market (Board and Lu, 2015).

Recently, Peters (2015) shows how tracking a buyer’s search history can be used in collusive pricing in a double auction environment. In Peters (2015), an on-line double auction is the default game that all sellers and buyers participate in and the competing mechanism is built on top of it. The environment is similar to the one for competing auctions, in that each seller has one unit of the homogeneous product equally valued at the same cost and each buyer needs only one unit with the private information on the valuation, which follows a probability distribution \( F \).

A seller can design a website to determine his ask price. Peters points out that a seller’s website can be understood as a mechanism. In the discovery phase, buyers are assumed to visit all the sellers’ websites. When buyers visit, sellers leave a cookie in their browsers that can be recovered when buyers revisit their websites. In the following exploratory phase, buyers continue to browse the sellers’ websites if they want. For example, if there is a deviating seller, buyers revisit all non-deviators to check if they lowered their ask prices as well. When a buyer revisits a seller’s website, the seller knows that the buyer has visited before because of the cookie. Therefore, whether or not a buyer revisits is simply a binary message of either 1 or 0. A seller can design a website that determines his ask price as a function of the number of buyers who revisit the site. It is simply an automated program. For example, a seller’s price is equal to some price above their cost if the number of buyers who revisit is less than some threshold, but it goes down to a very low level, say zero if that number
is above the threshold. Finally, sellers’ ask prices and buyers’ bidding prices are submitted in the double auction. This type of pricing based on cookies allows sellers to maintain a price above their cost because a seller’s deviation can induce buyers to revisit other sellers’ websites.

The applications mentioned in this section can also be applied to on-line markets because only the binary messages are needed to convey the relevant market information and all the results are preserved. For example, in competing prices, a seller’s website is programmed to set his price as a function of the number of buyers who revisits the site. In competing auctions, a seller’s website can be programmed in a way that his reserve price is a function of buyers who revisit. If the sellers impose entry fees together with the competitive per-unit price in competing ski-lift pricing, their website can be programmed so that the profile of entry fees charged is a function of the number of the buyers who revisit. Through the discovery and exploratory phases of the buyers’ search process, the reserve prices or the entry fees are determined and the buyers finally select one of the sellers for trading.

Furthermore, we proved that in various competing mechanism problems, a deviator cannot gain in any (truthful) continuation equilibrium upon deviation to any arbitrary mechanism once sellers and buyers play the competitive equilibrium in a restricted set of DIC mechanisms. Therefore, the competitive equilibrium can be used as a robust punishment that makes a deviator no better off in any (truthful) continuation equilibrium upon any deviation to any arbitrary mechanism. Also, we fully decentralize the competing mechanism game in the sense that we do not require a default game such as a double auction where the competing mechanism game is built on.

5 Conclusion

Ironically, it is quite easy to maintain implicit collusion in a large market because the competitive equilibrium terms of trade can serve as the robust punishment when there is a deviating seller. We believe that our approach can also be applied to other problems. For example, Norman (2004) considers public good provision with exclusion for a single mechanism designer. Using the results from this paper we can also consider competing public good provision with exclusion where buyers eventually select one seller for a public good.

This paper is based on the private value environment in the sense that

\[16\]Any positive number of revisiting buyers works for the threshold in Peters (2015) because buyers do not revisit if there is no deviators in the discovery phase. In practice, sellers may want to set up the threshold to a certain number, say a majority of the initial visitors.
each agent’s type affects only her payoffs. It is not difficult to imagine an interdependent value environment where an agent’s payoff depends on other agents’ types as well. In this case, one can consider a set of ex-post incentive compatible (EPIC) direct mechanisms (Bergemann and Morris 2005) to punish a deviator. The property of EPIC also does not depend on the endogenous distribution of the number of participating agents given that the participating agents report their true types. Therefore, one can always fix truthful type reporting to non-deviators.

Finally, it is important to see the relation with the literature on repeated games. Repeated games typically restrict what sellers can do in terms of contracting and then characterize the set of equilibrium allocations. We can apply the notion of robust equilibrium with DSIP without restricting mechanisms that principals can offer in the repeated games where principals repeatedly offer one-shot mechanisms over time. Suppose that a principal’s deviation is observable by the other principals at the end of the period and principals are sufficient patient but agents are myopic. Given mechanisms offered by principals, myopic agents play the stage game exactly the same way they would play in the one-shot game. By using this implication of how myopic agents play the stage game, we can indeed show that the set of robust equilibrium allocations supportable with DSIP in the repeated game with patient principals and myopic agents is identical to that in the one-shot game formulated in our paper.\textsuperscript{17}

When a principal’s deviation is observable by the other principals at the end of the period, principals do not need to use DDMs on the path in the repeated game to see whether or not there is a deviator. Only direct mechanisms are sufficient on and off the path to support a robust equilibrium allocation in the repeated game. For example, sellers only need to post prices in competing prices; post reserve prices in competing auctions; post menus of quantity and payment based on competitive unit price and entry fees in competing ski-lift pricing. If a seller deviates, non-deviating sellers lower their posted prices to the competitive level in competing prices; lower their reserve prices down to their costs in competing auctions; eliminate entry fees in competing ski-lift pricing. Our paper shows that the observability of a principal’s deviation in the repeated game can be replaced by the mechanism design (i.e., DDMs) in the one-shot game to induce agents to reveal a competing principal’s deviation as

\textsuperscript{17}To my knowledge, there are no studies in the repeated competing mechanisms except Ghosh and Han (2016). Their model is different from that in our paper in that an agent can trade with all principals. They showed that if agents are also patient, principals can force them to punish a deviating principal more severely even if it does not constitute a continuation equilibrium in a one-shot game. See Ghosh and Han (2016) for details on how it changes the set of equilibrium allocations and incentive compatibility in the repeated games where agents can trade with all principals and both agents and principals are patient.
well as their payoff types

References


