A COALITION-FORMATION APPROACH TO EQUILIBRIUM FEDERATIONS AND TRADING BLOCS

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Abstract. We develop a model in which states may choose to form coalitions to capture efficiency gains from policy coordination. Joining a coalition entails setting the policy variable to maximize the coalition’s aggregate payoff at a Nash equilibrium against non-members, and to commit to a transfer scheme to share the gains. With two states, the unique equilibrium structure is complete federation; with more than two states, incomplete federation can be the unique equilibrium. Interpreting this result in terms of customs unions, the trend to-trading bloc formation may be equilibrium behavior even with cooperation and transfers within customs unions.

JEL: F02, F13, H7

In many areas of economics, for example, international trade, international finance and local public economics, one compares allocations arising from independent policy decisions by governments with allocations that may be achieved through a coordina-

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tion of policies. The general conclusion points to potential gains from policy coordination since the policies are often strategically linked. A natural question, then, is: would a policy-coordinating federation of governments evolve to exploit the gains? The question is especially relevant given the growing number of initiatives towards policy-coordinating trading blocs and federations such as the North American Free Trade Agreement (NAFTA), the Association of South-East Asian Nations (ASEAN), and the European Union (EU), to name a few.

We believe that an answer to this question should be based on an explicit model of how states choose coalition partners. Such models are almost totally absent from the standard literature in the fields to which we have referred. In this paper we begin with a standard capital tax competition model and show how an equilibrium federal structure may arise from the strategic behaviour of initially unattached states.

One of the oldest strands in the trade literature, the theory of customs unions, has long been concerned with the removal of barriers to trade through formation of coalitions.¹ Much of this literature, however, treats the structure of coalitions as exogenously given and thus does not address the question of whether forming a particular coalition is in the interest of member states. Two notable exceptions are Murray C. Kemp and Henry Y. Wan (1976) and Raymond Reizman (1985). Kemp and Wan showed that a larger customs union can potentially make all of its members better off. This suggests that a global customs union—in other words, free trade—would dominate any other union for every state. Full exploitation of the gains from policy coordination requires that each state can commit to not using tariffs against other members of the union and to a transfer mechanism to share the gains. Reizman was one of the first to examine the structure of stable customs unions. He used the core

¹See Jaime DeMelo et al. (1992) for a recent review.
as a solution concept but precluded the possibility of inter-state transfers and thus concluded that world-wide union might not be a stable outcome.\textsuperscript{2} A strand in the literature of international finance that studies gains from coordinating monetary policies to internalize externalities associated with internationally transmitted inflation has lately attracted much attention. On this topic, the paper by Alessandra Casella (1992) is the one most closely related to our approach of policy coordination through coalition formation.\textsuperscript{3}

Although coalitions are the primitive in cooperative game theory, the standard literature in this area also largely takes the structure of coalitions as exogenous and concentrates on the question of how the total worth of a coalition is allocated to its members. Again, there are a number of notable exceptions. Indeed, the classic treatise of John von Neumann and Oskar Morgenstern (1944) itself contains a model of coalition formation. The important work of Sergiu Hart and Mordecai Kurz (1983) is similar in spirit and provides an explicit modelling of coalition formation. The topic has received greater attention of late: see, for example, Francis Bloch (1992), Debraj Ray and Rajiv Vohra (1992), Cheng-Zhong Qin (1995), and Sang-Seung Yi (1995).

The context in which we study the problem of federation formation is a very simple

\textsuperscript{2}Consider a two-state example of Reizman’s model. If the two states agree to a customs union, because of the absence of transfers, they consume the particular allocation associated with free trade. But if one of the states does better with a tariff war (here large states win tariff wars) then using Reizman’s model we would conclude that the world-wide union would not be the outcome even though strict Pareto improvements are possible, if transfers were allowed. In our paper we admit the possibility of transfers. Another strand of the customs union literature (e.g. Kyle Bagwell and Robert W. Staiger (1993)) has argued against allowing binding commitment to a customs union. While the study of self-enforcing tariff arrangements between states is a useful approach we would argue it should not be the exclusive approach. This strand of the literature does typically allow cooperation within countries and today’s countries are just yesterday’s coalitions. Thus the level at which it is sensible to allow commitments is unclear and the study of the implications of allowing commitment at different levels seems justified. We follow the traditional literature and assume commitments within coalitions are possible.

\textsuperscript{3}In particular see the longer version of this paper, Casella (1990).
one-good model of capital tax competition. There is inter-state trade of mobile capital for the consumption good. Each state chooses a tax rate to levy on locally employed capital to manipulate the terms of trade. The non-cooperative choice of taxes leads to inefficient Nash equilibria with too little trade. Therefore, there are potential gains to coordinating tax policies.

To allow states to capture the benefits of policy coordination, we suppose that any group of states may choose to form a federation (equivalently, a coalition) and arrange to share the gains. To join a coalition is to make a binding commitment to behave cooperatively with other members of the coalition. Specifically, it entails setting capital taxes against non-members at rates that maximize the coalition's aggregate consumption at a Nash equilibrium and splitting the coalition's aggregate consumption in accordance with a sharing rule. The rule we focus on for the most part involves sharing the gains equally. Thus, corresponding to each federation structure, that is, an alignment of the states into coalitions, non-cooperative Nash equilibrium in capital taxes yields an aggregate consumption for each coalition and the cooperative sharing rule assigns an allocation of the aggregate to each coalition member. This induces a preference ranking for each state over all possible coalition structures. Following the ideas of von Neumann and Morgenstern (1944) and Hart and Kurz (1983), we think of each state formulating a partnership plan, that is, a list of states it would like to join in a coalition. A profile of partnership plans by the states determines a coalition structure. If there is unanimous agreement among a group of states—each would like to join precisely the others in a coalition—we assume such a coalition indeed forms. In the event that the partnership plans of states are not all mutually compatible, we explore alternative ways of reconciling them. We regard a profile of partnership plans by the states as an equilibrium if no coalition of states, taking the plans of its
complement as fixed, can fashion a profitable deviation for each of its members that is itself immune to further deviations by subsets of the deviating coalition. Finally, we define a federation structure as an equilibrium if it is the outcome of an equilibrium profile of partnership plans by states.

Driven by the recent trend towards formation of trading blocs—such as the EU and the NAFTA—the viability of global free trade, promoted through the GATT and the WTO, has emerged as an important and controversial issue. In this regard, we ask the following question. In a model in which full efficiency gains from policy coordination can be achieved only by a coalition of all states, in which forming coalitions is not costly per se and in which mechanisms to share the efficiency gains through interstate transfers exist, should one necessarily expect the emergence of global free trade, that is, the formation of the grand coalition, in equilibrium?

As long as there are only two states, we show that complete federation is the unique equilibrium structure. As soon as we move beyond a world with only two states, we show that the grand coalition need not be the unique equilibrium coalition structure. Indeed, we provide an example in which a coalition structure other than the grand coalition is the unique equilibrium structure. Further, we show that this result is rather robust to modeling specifications. Although the grand coalition always captures all efficiency gains and accordingly there are points on the grand coalition’s consumption possibility frontier which are Pareto improvements over any single alternative coalition structure, there may be no point that constitutes a simultaneous Pareto improvement over consumption bundles associated with every alternative structure. Thus even with commitment to coalitions and transfers, there are natural impediments to a successful implementation of global free trade.

Section I presents the model of capital tax competition which provides the back-
drop for our study. Section II serves as an overview of the model, which comprises three distinct stages: the game of coalition formation; capital tax competition; and bargaining within a coalition over consumption shares. Sections III, IV and V each describe a stage and section VI sets out the results. The paper concludes with a discussion of some limitations and possible extensions of the approach presented in the paper. We emphasize that the modular structure permits easy substitution of the capital tax component with a model of multi-commodity trade or international finance or public good provision. Accordingly, the general framework of the paper can be applied to many different contexts.

I. The Backdrop: A Model of Capital Tax Competition

The backdrop of our study is a simple model of capital tax competition among a set of states, $N = \{1, \ldots, n\}$. Each state $i$ in $N$ has a given population, $L_i$. Every individual in state $i$ is endowed with one unit of labour and $K_i/L_i$ units of capital. Labour is immobile and must be employed in the state of residence; capital is costlessly mobile across states. A single and identical good is produced by perfectly competitive firms in each state $i$ in $N$, using capital and labour as inputs, via a technology described by a twice continuously differentiable, 1-homogeneous production function, $F_i(\cdot,\cdot)$, with diminishing marginal products. Each state $i$ may levy a per-unit tax $t_i$ (or subsidy) on capital employed in the state; the state budget is balanced by a head tax $\tau_i$ (or subsidy) on each resident. Individuals sell their capital in the state offering the highest after-tax return on capital. We assume that all labour and capital are fully employed and that some capital is employed in every state.

Since labour is immobile, the amount of labour employed in state $i$ is simply $L_i$.

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4For related models of tax competition, see Koichi Hamada (1966) and David E. Wildasin and John D. Wilson (1991).
Let $K_i$ denote the amount of capital employed in state $i$; under full employment,

(1) \[ \sum_{i=1}^{n} K_i = \sum_{i=1}^{n} \bar{K}_i. \]

Since production in each state takes place under competitive market conditions, labour and capital receive their marginal products, denoted $F_i^L$ and $F_i^K$ respectively; moreover, absence of opportunities for profitable arbitrage entails equalization of after-tax rate of return on capital across states. Let $\rho$ denote the common after-tax return on capital; then, we have

(2) \[ \rho = F_i^K - t_i, \quad i = 1, \ldots, n. \]

In what follows, we shall assume that, for any given profile $t = (t_1, \ldots, t_n)$ of tax rates on capital prevailing in the $n$ states, the system of equations (1) and (2) uniquely determine $K_1(t), \ldots, K_n(t)$ and $\rho(t)$.

The objective of every state $i$ in $N$ is to set capital tax rate $t_i$ to maximize the aggregate consumption of state $i$, defined as the total of net-of-tax labour and capital income of all its residents (in units of the single good). Since $F_i$ is 1-homogeneous,

\[ F_i^L(K_i(t), L_i)L_i = F_i(K_i(t), L_i) - F_i^K(K_i(t), L_i)K_i(t) \]

so that the aggregate consumption to state $i$ is given by:

(3) \[ C_i = F_i(K_i(t), L_i) - F_i^K(K_i(t), L_i)K_i(t) + \rho(t)\bar{K}_i - \tau_iL_i. \]

Using (2), (3) may be expressed as

(4) \[ C_i = F_i(K_i(t), L_i) - \rho(t)(K_i(t) - \bar{K}_i) - (t_iK_i(t) + \tau_iL_i). \]

But for each state $i \in N$, the state budget is balanced so that $t_iK_i(t) + \tau_iL_i = 0$. Therefore, using $I_i$ as a shorthand for $(K_i(t) - \bar{K}_i)$, the net capital imports into state $i$, we may write (4) as

(5) \[ C_i = F_i(K_i(t), L_i) - \rho(t)I_i(t). \]
Efficiency—that is, total product maximization—requires equalization of marginal products of capital across states which, in turn, requires—see (2)—that each state \( i \) in \( N \) set the same per-unit tax rate \( t_i \) on the capital employed in the state. If the states were to choose capital tax rates non-cooperatively, the profile of choices at a Nash equilibrium would be inefficient as long as some inter-state movement of capital were necessary to equalize marginal products of capital across states. As we show later in the paper, an increase in capital tax rate \( t_i \) by a state leads to a decline in the equilibrium value of \( \rho(t) \). Thus, to effect a favourable terms of trade, an importer of capital has an incentive to tax capital (set \( t_i > 0 \)) and an exporter of capital has an incentive to subsidize capital (set \( t_i < 0 \)): both result in a reduction in the volume of trade below the efficient level.

In such a setting, therefore, there are gains to coordinating tax policies. Could one expect to see the emergence of a stable cooperative arrangement among the states to exploit the potential gains? The object of this paper is to develop a model to try to answer this question.

II. An Overview

We allow any group of states to band together in coalitions, coordinate their choices of capital tax rates and commit to an arrangement to share the gains. Formally, a coalition (or, a federation) is simply a nonempty subset of the set of states \( N \); a coalition structure (or, a federation structure) is a partition of the set of states, denoted \( B = \{T_1, \ldots, T_m\} \). \( B \) will stand for the set of all coalition structures; the coalition \( N \) consisting of all states will be called the grand coalition.

Given an alignment of the \( n \) states into a federation structure \( B = \{T_1, \ldots, T_m\} \) and a profile of capital tax rates \( t = (t_1, \ldots, t_n) \) set by the states, the aggregate
consumption available to federation $T_j$, $C_{T_j}(B)$, is obtained by summing (4) over all $i$ in $T_j$. With a balanced federal budget, $\sum_{i \in T_j} (t_iK_i(t) + \tau_iL_i) = 0$. Thus,\footnote{When individual state budgets do not balance there are transfers between states within a federation. The head tax instruments allow any distribution of a federation's aggregate consumption among member states.}

(6) \[ C_{T_j}(B) = \sum_{i \in T_j} \left( F_i(\{K_i(t), L_i\} - \rho(t)I_i(t) \right). \]

We suppose that to join a federation is to make a binding commitment to cooperative behaviour with other member states: specifically, (a) to set capital taxes against non-members at rates that maximize the federation's aggregate consumption at a non-cooperative Nash equilibrium; (b) to split the federation's aggregate consumption according to the dictates of a particular cooperative bargaining solution, specified in Section III below. Thus, to any grouping of states into a coalition structure $B$, there corresponds a Nash equilibrium profile of tax rates set by the states and a resultant aggregate consumption available to each coalition of states. Let $C_{T_j}^*(B)$ denote the Nash equilibrium level of aggregate consumption for coalition $T_j$ in the structure $B$. The coalitional bargaining rule, to which the coalition members are pre-committed, then determines the allocation of $C_{T_j}^*(B)$ among the members of $T_j$. We are interested in characterizing the "equilibrium" coalition structures that arise as a result of a set of initially unattached states strategically choosing coalition partners in a sense made precise in Section V below.

It is conceptually helpful to think of the model as unfolding in three distinct stages. In the first stage, a coalition structure is determined through forward-looking states choosing coalition partners. In the second stage, each coalition sets a capital tax rate against non-members; this determines the Nash equilibrium level of aggregate
consumption available to each coalition. Finally, in the third stage, the aggregate consumption is divided among the coalition members according to a coalitional bargaining rule. Thus, the second and third stages induce, for each state, a preference ordering over possible coalition structures upon which strategic partnership plans can be based. We now turn to a detailed description of each of the stages, spelt out in the next three sections, starting with the third stage.

III. Bargaining within a Given Coalition

We begin with the stage at which the states are already aligned into coalitions—thus, the coalition structure $B = \{T_1, \ldots, T_m\}$ is given; capital taxes have been set by each state; factors of production have adjusted to the taxes; output has been produced; and the Nash equilibrium level of aggregate consumption available to each coalition $T_j$ in $B$, $C^*_j(B)$, is already known.

The problem at this stage is the classical bargaining problem: how to divide the aggregate worth of a given coalition among its members. This, of course, has been a central question in cooperative game theory since the inception of the subject and a great many solution concepts have been proposed. We restrict attention to solution concepts that are guaranteed to be nonempty and single-valued: in other words, those that always prescribe a unique payoff to each member of the given coalition. This is useful in ensuring that states have a well-defined preference ordering on the basis of which coalition partners can be chosen. Examples of solution concepts in cooperative game theory that are always nonempty and single-valued are the Nash bargaining solution, the Shapley value and the nucleolus. Examples of well-known solution concepts in cooperative game theory that do not meet these requirements are the core and the bargaining sets: the core may be empty or quite large; the bargaining
set, although always nonempty, is even larger than the core.  

In general, by a coalitional bargaining rule we mean a function \( \phi \) that assigns to any alignment of states into a coalition structure \( B = \{T_1, \ldots, T_m\} \) an allocation of aggregate consumption \( \phi_i(B) \) for every \( i \) in \( T_j \) such that

\[
\sum_{i \in T_j} \phi_i(B) = C_{T_j}^*(B) \quad \text{for every } T_j \in B.
\]

We often refer to \( \phi_i(B) \) as the payoff of state \( i \) in coalition \( T_j \) under the bargaining rule \( \phi \). We suppose that joining a coalition entails an irrevocable commitment to a particular bargaining rule \( \phi \). What would be a sensible modelling choice for the rule \( \phi \) in this context?

The rule we focus on for the most part dictates that members equally share the surplus attributable to their cooperation in forming the coalition, taking as given the structure of complementary coalitions. To formalize this rule, let the coalition structure \( B \) and \( T_j \in B \) be given and let \( T_j = \{i_1, \ldots, i_l\} \). Denote by \( \hat{B} \) the coalition structure in which the individual members of \( T_j \) are singletons while the other states are organized in coalitions as given in \( B \): that is,

\[
\hat{B} = \{T_1, \ldots, T_{j-1}, \{i_1\}, \ldots, \{i_l\}, T_{j+1}, \ldots, T_m\}.
\]

Denote by \( C_i^*(\hat{B}) \) the Nash equilibrium consumption level that a member \( i \) of \( T_j \) will have in the coalition structure \( \hat{B} \). One notion of the surplus attributable to cooperation among members of \( T_j \) then is:

\[
C_{T_j}^*(B) - \sum_{i \in T_j} C_i^*(\hat{B}).
\]

\(^6\)A lucid and elegant textbook treatment of cooperative solution concepts we have referred to may be found in Martin J. Osborne and Ariel Rubinstein (1994).
From the point of view of the members of $T_j$, (8) represents the total gain achievable by forming a coalition, taking as given the structure of complementary coalitions. Splitting this surplus equally among coalition members amounts to adopting the particular bargaining rule $\phi^*$ which prescribes for $i$ in $T_j$ the payoff

$$
\phi^*_i(B) = C^*_i(\hat{B}) + \frac{1}{|T_j|} \left( C^*_{T_j}(B) - \sum_{i \in T_j} C^*_i(\hat{B}) \right),
$$

where $|A|$ represents the cardinality of a set $A$.

The bargaining rule $\phi^*$ is easily seen to be related to the literature on axiomatic bargaining theory. In the terminology of this literature an $l$-agent bargaining problem consists of two objects: (i) a set of feasible payoffs in $\mathbb{R}^l$ and (ii) a disagreement point, a payoff vector in $\mathbb{R}^l$. If we regard the allocation of $C^*_{T_j}(B)$ among the members of $T_j$ as a bargaining problem in which the set of feasible payoffs is the one-point set $\{C^*_{T_j}(B)\}$ and the disagreement point is the vector $(C^*_i(\hat{B}))_{i \in T_j}$, the rule $\phi^*$ is nothing but the $l$-person Nash bargaining solution. Indeed, it is easy to show that for any transferable utility bargaining problem, any solution satisfying Nash’s axioms of independence of utility origins, efficiency and symmetry recommends equal division of gains from cooperation—see Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green (1995), Exercise 22.F.1—and thus recommends (9) for the simple bargaining problem defined above.

We emphasize that in our model a commitment to abide by the coalitional bargaining rule is a precondition of joining a coalition. Although we used the terminology of axiomatic bargaining theory in the preceding paragraph, we do not mean to suggest that a coalition splintering apart because of “disagreement” among its members is a possibility at this stage of the model: after all, the coalition structure is already given at this point. Rather, the Nash equilibrium consumption levels $C^*_i(\hat{B})$ for $i$ in $T_j$ are
only used as part of a notional calculation in arriving at a measure of the surplus attributable to cooperation among the members forming the coalition.

In what follows, we write $\phi$ to indicate a general coalitional bargaining rule; we write $\phi^*$ if we specifically refer to the Nash bargaining solution given by (9) above.

IV. Nash Equilibrium in Capital Tax Competition

Next, we consider the stage at which the coalition structure $B = \{T_1, \ldots, T_m\}$ is given and the capital tax rates, $t = (t_1, \ldots, t_n)$, are about to be chosen. By joining a coalition $T_j$, a state is committed to setting a tax rate that maximizes the coalition’s aggregate consumption, $C_{T_j}(B)$, at a Nash equilibrium against non-members.

In the Appendix, we characterize the profile of tax rates $(t_i)_{i \in T_j}$ that would be dictated by a federal government of coalition $T_j$ for its member states with a view to maximizing the coalition’s aggregate consumption $C_{T_j}(B)$ at a Nash equilibrium against non-members. We show that each coalition’s optimal choice entails each member state choosing the same capital tax rate: in particular,

\begin{equation}
    t_i = \frac{I_{T_j}}{\sum_{h \notin T_j} (1/F^K_h)} \quad \text{for every } i \in T_j,
\end{equation}

where $F^K_i = \partial F^K_i / \partial K_i < 0$. This leads to an efficient allocation of capital within the coalition and to competing coalitions facing a common strategic policy. In effect, each coalition functions as a customs union.

The system of equations (1), (2) and (10) characterize the equilibrium values of $t$, $K_1(t), \ldots, K_n(t)$ and $\rho(t)$. For each coalition $T_j$ in $B$, the Nash equilibrium levels of aggregate consumption, $C_{T_j}^+(B)$ is then determined by (6). Finally, the payoff of each state $i$ in $T_j$, $\phi^*_i(B)$, is determined by the coalitional bargaining rule $\phi^*$ given by (9).

We also show in the Appendix that if each state, having already joined a coalition—and thereby having committed to the bargaining rule $\phi^*$—were to choose its capital
tax rate with a view to maximizing only the well-being of its own citizens, its choice would coincide with the rate given in (10). Thus, in contrast to the customs union literature, the coalition’s actions are invariant to which state or set of states controls the government. It is in each coalition member’s own interest to maximize the coalition’s aggregate consumption: maximizing $C_{T_j}(B)$ maximizes $\phi^*_i(B)$.

Let the common value of $t_i$ in (10) for members of $T_j$ be denoted by $t_{T_j}$. Efficiency requires equalization of capital taxes: each coalition $T_j$ must choose the same tax rate $t_{T_j}$. By (10), $t_{T_j}$ will have the same sign as $I_{T_j}$. Thus, if one coalition imports capital then another must export capital and they would have unequal capital taxes: the importer taxes capital and the exporter subsidizes capital. Nash equilibrium generates inefficiency as long as efficiency requires any movement of capital across coalitions. Similar results are of course familiar from the optimal tariff literature.\footnote{We would also like to note that the addition of import tariffs to this model adds nothing to our analysis. In the presence of capital taxes, tariffs are redundant because each state already has an instrument to manipulate its terms of trade.}

A special case in which efficiency does not require trade across coalitions occurs when all states join to form the grand coalition, that is, when $B = \{N\}$. In that case we have global free trade and efficiency.

\section{V. Coalition Formation}

We now start from a status quo of a set of unattached states and model how states, acting in their own self-interest, might choose to align themselves into coalitions at the very outset. The model of coalition formation we adopt dates back to the classical work of von Neumann and Morgenstern (1944) and the more recent work of Hart and Kurz (1983).

First, recall that for any given coalition structure $B = \{T_1, \ldots, T_m\}$, capital tax competition among the coalitions in $B$, as described in Section IV, would determine
the Nash equilibrium level of aggregate consumption available for each coalition \( T_j \) in \( B \); sharing the total among the member states according to the coalitional bargaining rule, as described in Section III, would determine a payoff for each member state \( i \) of each coalition \( T_j \). Thus, a state, in cognizance of the subsequent stages, would have a preference ordering over possible coalition structures. We use these preference orderings to construct a game in strategic form.

It is worth elaborating on our (unmodelled) stylized view of the process that results in the formation of a coalition structure. We view the group of independent states engaging in non-binding pre-play communication during which possible options for coalition formation are weighed and potential partners sought. Eventually, each state comes to formulate a plan for joining a set of partners. A strategy of state \( i \) will be identified with a partnership plan by state \( i \): it is a choice of a coalition to which \( i \) wants to belong. Formally, a strategy for state \( i \) is simply a subset \( S_i \) of \( N \) with the property that \( i \in S_i \). A combination of choices of participation plans or strategies, one for each state, \( s = (S_1, \ldots, S_n) \), will be referred to as a profile of partnership plans or a strategy profile.\(^8\) The set of all partnership plans for state \( i \) will be denoted by \( S_i \); \( S = \bigcup_{i \in N} S_i \) will stand for the set of all profiles of partnership plans.

How any given profile of partnership plans \( s \in S \) gets reconciled into a resultant coalition structure is summarized by a function, \( \psi : S \to B \), that assigns to any \( s \in S \) a coalition structure \( B = \psi(s) \). We call the function \( \psi \) the coalition structure rule. Informally, the rule \( \psi \) is meant to capture the states' beliefs, assumed to be commonly held and correct. The question, then, is: what is a sensible modelling choice for the function \( \psi \)?

\(^8\)Throughout, we shall use the two terms partnership plan and strategy synonymously. Strategy is the formal name; a partnership plan should be thought of as the intuitive idea that it is trying to capture in our context.
Let us say that a group of states $T_j$ have a \textit{unanimous} partnership plan if $S_i = T_j$ for every $i \in T_j$. Note that unanimity among a group requires not only that states in the group have an identical plan, but also that the plan calls for each member in the group to join precisely the other members in the group.\footnote{A coalition $T_j$ with a unanimous partnership plan is called a \textit{ring} by von Neumann and Morgenstern (1944).} A minimal property we demand of any coalition structure rule $\psi$ is that it be \textit{unanimity-preserving}: should a group $T_j$ of states be unanimous on a partnership plan, coalition $T_j$ indeed forms.

In general, it is perfectly possible of course for state $i$ to plan to belong to a coalition $T_j$ that includes state $k$ but for state $k$ to have no intention of joining some state $l$ in $T_j$. How are such mutually incompatible participation plans to be reconciled? An example may be helpful.

\textbf{Example.} Suppose $N = \{1, 2, 3, 4, 5, 6\}$ and

$$
\tilde{S}_1 = \tilde{S}_2 = \{1, 2\}, \quad \tilde{S}_3 = \tilde{S}_4 = \{3, 4, 5\}, \quad \tilde{S}_5 = \{5\}, \quad \tilde{S}_6 = N;
$$

let $\tilde{s} = (\tilde{S}_1, \tilde{S}_2, \tilde{S}_3, \tilde{S}_4, \tilde{S}_5, \tilde{S}_6)$.

The requirement that $\psi$ preserve unanimity is quite weak. In the example, it entails only that, in the coalition structure $\psi(\tilde{s})$, states 1 and 2 form the coalition $\{1, 2\}$ and state 5 remain as a singleton.\footnote{Note that the definition of having a unanimous plan is trivially satisfied by a state choosing to remain on its own.} It does not restrict how states 3, 4 and 6 are grouped.

In most contexts involving federation, it would seem reasonable to require also that no state be coerced into a coalition with other states it had not planned to join. This means not forcing either state 3 or 4 into a partnership with state 6. The choice on states 3 and 4 is perhaps less clear-cut. Is it more reasonable to entertain a coalition structure in which states 3 and 4 are singletons—because each chose state 5 as a partner—but state 5 is not available as a partner—or to suppose that they would form...
a coalition because their plans coincide? Given that our premise is a set of singleton states, we find it more natural to suppose that a multi-state coalition can form if and only if there is unanimous consent among the members of the coalition in question.

Call $\psi^* : S \rightarrow B$ the strict unanimity rule if $\psi^*(s)$ is the partition induced by states having unanimous plans forming coalitions. For the profile $\bar{s}$ in the example,

$$\psi^*(\bar{s}) = \{\{1, 2\}, \{3\}, \{4\}, \{5\}, \{6\}\}.$$

Thus, a group of states that fail to achieve unanimity with respect to a partnership plan retain their status quo ante position and remain as singleton coalitions. In other words, each state $i$ is either in its planned coalition, $S_i$, or as it had started—as a singleton.

We recognize that there are contexts in which the strict unanimity rule will not seem compelling. In particular, one may argue that, for some environments, $\psi^*$ prescribes too fine a partition and a rule that gives rise to a coarser structure is called for.\(^\text{11}\)

Let us say that a group of states $T_j$ have a similar partnership plan if, for every $i \in T_j$, $S_i = T$ for some $T \subseteq N$. Note that here we do not insist that $T = T_j$; similarity is a weaker requirement than strict unanimity. Call $\hat{\psi} : S \rightarrow B$ the similarity rule if $\hat{\psi}(s)$ is the partition induced by states who choose a similar partnership plan forming coalitions. For the example above,

$$\hat{\psi}(\bar{s}) = \{\{1, 2\}, \{3, 4\}, \{5\}, \{6\}\}.$$

Among the two choices for the rule $\psi$, $\psi^*$ and $\hat{\psi}$, it is clear that $\hat{\psi}$ is more cohesive. Indeed, given an arbitrary profile $s$ of partnership plans, in a sense, $\psi^*(s)$ selects the finest plausible unanimity-preserving coalition structure while $\hat{\psi}(s)$ selects the

\(^{11}\text{A partition } B \text{ is called finer than a partition } B' \text{ if for any } T_j \in B \text{ there is } T'_k \in B' \text{ such that } T_j \subseteq T'_k.\)
coarsest plausible structure. Note also that $\psi^*(s) = \hat{\psi}(s) = \{N\}$ if and only if $s$ has every component $S_i = \{N\}$ for each $i$ in $N$. Under either the strict unanimity rule or the similarity rule, the grand coalition forms only as a consequence of a unanimous partnership plan shared by all states. In what follows we write $\psi$ to indicate a general coalition structure rule; we write $\psi^*$ or $\hat{\psi}$ if we want to refer specifically to the strict unanimity rule or the similarity rule.

We now have a well-defined game in strategic form. The players are the set $N$ of states; the set of strategies available to each player $i$ in $N$ consists of all possible partnership plans, $S_i$; every strategy profile $s$ induces a coalition structure $\psi(s) = \{T_1, \ldots, T_m\}$ through the coalition structure rule, and thus a payoff for each player $i$ in $N$ of $\phi_i(\psi(s))$ through the coalitional bargaining rule. We call the game the coalition formation game.

We want to identify a coalition structure $B$ as an “equilibrium” structure if $B = \psi(s)$ for an “equilibrium” strategy profile $s$ for the coalition formation game. The question now is: what would be an appropriate equilibrium concept for the model?

The solution concept most commonly invoked for games in strategic form is surely the Nash equilibrium. Recall that a strategy profile is a Nash equilibrium if no player has a unilaterally profitable deviation. For our purpose, however, Nash equilibrium is far too weak a solution concept. For instance, $s = (S_1, \ldots, S_n)$ with $S_i = \{i\}$, $i = 1, \ldots, n$, is a Nash equilibrium of the coalition formation game for any unanimity-preserving coalition structure rule $\psi$ no matter what coalitional bargaining rule $\phi$ is in effect. No state $i$, taking as given the strategy choices of the other $(n - 1)$ states, can affect the resultant coalition structure $\psi(s) = \{\{1\}, \ldots, \{n\}\}$ and hence its pay-off. Therefore, the singleton coalition structure $\{\{1\}, \ldots, \{n\}\}$ is a Nash equilibrium structure irrespective of the payoff functions in the game.
Clearly, for a model of coalition formation such as this one, one should allow a group of states to coordinate on a multilateral deviation if such a deviation were to make each deviating state better off. We are thus led to a subset of Nash equilibria which ensure that equilibrium profiles are immune to individual as well as joint deviations. An especially attractive refinement in this class is the concept of coalition proof Nash equilibrium (CPNE), due to B. Douglas Bernheim, Bezalel Peleg and Michael Whinston (1987). Roughly, a strategy profile is coalition proof if no set of states, taking the strategies of its complement as fixed, can fashion a profitable deviation for each of its members that is itself immune to further deviations by subsets of the deviating coalition. We refer the reader to the original article for a formal definition.

We call a coalition structure $B$ an equilibrium coalition (or, federation) structure if $B = \psi(s)$ for a CPNE strategy profile $s$ for the coalition formation game.

VI. Equilibrium Federation Structure

In the face of the recent trend towards regional trading blocs, viability of global free trade has emerged again as an important issue—not simply because it is a matter of obvious practical significance but also because it invites a re-examination of some of our theoretical models. In much of the theoretical literature the usual approach has been to treat the structure of coalitions—such as the structure of trading blocs—as given. The usual conclusion is that there are potential gains to be exploited by coordinating policies across all coalitions. This may seem to suggest that, absent impediments per se to forming large coalitions and arranging transfers, the formation of the grand coalition—that is, global free trade—is the likely outcome. As stressed in the introduction, we believe that such analysis should be conducted in the context of an explicit model of an equilibrium coalition structure.
Accordingly, in the context of the current model, the question we focus on is: if the grand coalition can capture full efficiency gains from policy coordination and if the gains can be shared through inter-state transfers, should one necessarily expect that the grand coalition would form?

If there are only two states and efficiency requires inter-state trade, we record, in the observation below, that the model indeed yields the grand coalition as the equilibrium structure.

**Proposition 1.** Let the set of states be \( N = \{1, 2\} \) and suppose that efficiency requires inter-state trade. Let the coalitional bargaining rule be the Nash bargaining solution \( \phi^* \) and let the coalition structure rule \( \psi \) be any unanimity-preserving rule. The coalition structure \( \{\{1, 2\}\} \) is the unique equilibrium coalition structure.

**Proof.** By hypothesis, since efficiency requires inter-state trade, we have\(^{12}\)

\[
C^*_i(\{\{2\}\}) - (C^*_i(\{\{1\}\}, \{2\}\}) + C^*_2(\{\{1\}\}, \{2\}\}) > 0.
\]

(11)

By definition,

\[
\phi_i(\{\{1\}\}, \{2\}\}) = C^*_i(\{\{1\}\}, \{2\}\}) \quad i = 1, 2.
\]

Using the definition of the Nash bargaining rule \( \phi^* \), given in (9), we have

\[
\phi^*_i(\{\{1\}\}, \{2\}\}) < \phi^*_i(\{\{1, 2\}\}) \quad i = 1, 2.
\]

It is now straightforward to verify that the profile of partnership plans \( s = (S_1, S_2) \) with \( S_1 = S_2 = \{1, 2\} \) is the unique CPNE of the coalition formation game for any unanimity-preserving coalition structure rule. Thus, \( \{\{1, 2\}\} \) is the unique equilibrium coalition structure. \( \square \)

\(^{12}\)We write \( C_{12} \) as a short-hand for \( C_{\{1,2\}} \).
Remark 1. Although we used the Nash bargaining solution $\phi^*$ as the coalitional bargaining rule in Proposition 1, it is clear that the proposition would be true for any bargaining rule $\phi$ that does not allocate the entire gain in (11) to only one player.

The logic behind Proposition 1 is extremely simple. Since the grand coalition captures all efficiency gains, the aggregate consumption realizable under it is higher than that under the singleton structure. Since transfers are feasible, the gain can be shared so that it is rational for the states to join together in a federation.

As we move beyond a world with only two states, it still remains true that when efficiency entails trade across federations, the aggregate consumption under the grand coalition will dominate that under every alternative coalition structure. This, however, does not guarantee that the grand coalition is the unique equilibrium structure. Indeed, it is easy to construct examples in which a coalition structure other than the grand coalition is the unique equilibrium structure. Thus, global free trade, even in the absence of impediments to formation of coalitions and to the arrangement of inter-state transfers, may not represent an equilibrium outcome at all. We formally record this observation in the proposition below.

Proposition 2. Let $|N| > 2$, let the coalitional bargaining rule be the Nash bargaining solution $\phi^*$ and let the coalition structure rule $\psi$ be any unanimity-preserving rule. A coalition structure other than the grand coalition may be the unique equilibrium coalition structure.

Proof. An example suffices to establish the proposition. Let $N = \{1, 2, 3\}$ and

\[
F_i(K_i, L_i) = K_i^{1/4} L_i^{3/4}, \quad i = 1, 2, 3;
\]

\[
L_1 = L_2 = \frac{3}{200}, \quad L_3 = \frac{97}{100}, \quad \bar{K}_1 = \bar{K}_2 = \frac{1}{2}, \quad \bar{K}_3 = 0.
\]

(Thus, states 1 and 2 are identical and relatively capital-abundant.) Routine computation yields the aggregate consumption available to each coalition for all possible
coalition structures at the Nash equilibrium in capital-tax competition: they are reported below.
\[
C_i^*({\{1\}, \{2\}, \{3\}}) = 0.073605, \quad i = 1, 2; \quad C_3^*({\{1\}, \{2\}, \{3\}}) = 0.823475, \\
C_{12}^*({\{1, 2\}, \{3\}}) = 0.243399, \quad C_3^*({\{1, 2\}, \{3\}}) = 0.683545, \\
C_{13}^*({\{1, 3\}, \{2\}}) = 0.904136, \quad C_{23}^*({\{1, 3\}, \{2\}}) = 0.079229, \\
C_{23}^*({\{2, 3\}, \{1\}}) = 0.904136, \quad C_1^*({\{2, 3\}, \{1\}}) = 0.079229, \\
C_{123}^*({\{1, 2, 3\}}) = 1.
\]

(13) Applying the coalitional bargaining rule $\phi^*$, we obtain $\phi_i^* (B_i)$ for each $B_i$ in $B$: they are tabulated below.

Table 1 here

We shall prove that $B^* = \{\{1, 2\}, \{3\}\}$ is the unique equilibrium coalition structure under any unanimity-preserving coalition structure rule $\psi$.

Consider any strategy profile
\[
(14) \quad s^* = (S_1^*, S_2^*, S_3^*) \quad \text{with} \quad S_1^* = S_2^* = \{1, 2\}.
\]

Since $\psi$ is unanimity-preserving, $\psi(s) = B^*$ regardless of $S_3^*$. We show that any such $s^*$ is a CPNE; moreover, any CPNE $s$ must satisfy $\psi(s) = B^*$.

To see that any $s^*$ satisfying (14) is necessarily a CPNE, note from the table above that $B^*$ is precisely the coalition structure for which the payoff of each of states 1 and 2 is at a maximum. Therefore, states 1 and 2 cannot possibly have a profitable deviation, unilateral or multilateral. A unilateral deviation by state 3 is, then, the only remaining possibility. But, a unilateral deviation from $s^*$ by state 3 would have no effect on the resultant coalition structure since $\psi$ is unanimity-preserving. Therefore, $s^*$ is a CPNE.
Now, suppose $s = (S_1, S_2, S_3)$ is a CPNE. Then, we must have $\psi(s) = B^*$: otherwise, states 1 and 2 would have a profitable joint deviation to $S_1 = S_2 = \{1, 2\}$ giving rise to the coalition structure $B^*$. Moreover, since $B^*$ maximizes the payoff of each of states 1 and 2, such a joint deviation would be immune to further profitable deviations by either state. Therefore, a CPNE $s$ must satisfy $\psi(s) = B^*$. □

Remark 2. Since the strict unanimity rule $\psi^*$ and the similarity rule $\hat{\psi}$ are special cases of the class of unanimity-preserving rules, it follows from Proposition 2 that the grand coalition need not be an equilibrium structure under either of them.

Remark 3. The example shows that the unique equilibrium coalition structure may turn out to be the one that involves the largest efficiency loss, as does the structure $B^*$.

Remark 4. It is perfectly possible of course for the grand coalition to be an equilibrium structure. A key parameter in the model of capital-tax competition is the ratio of a state's endowment of capital to its endowment of labour and a key feature of example (12) is the asymmetry in the capital-labour ratio between state 3 and (the identical) states 1 and 2. To construct an example in which the grand coalition is an equilibrium structure, change the parameters in example (12) to reduce this asymmetry. In particular, let $\frac{K_1}{L_1} = \frac{K_2}{L_2} = \frac{33}{5}$ and $\frac{K_3}{L_3} = \frac{41}{50}$; keep all other aspects of the example unchanged. Now, one can show that the grand coalition is the unique equilibrium coalition structure under either the strict unanimity or the similarity rule, when the coalitional bargaining rule is the Nash bargaining solution. For states 1 and 2, the advantages of policy coordination with state 3 now outweigh the advantages of forming a capital monopoly. This may suggest that, in general, reducing asymmetry facilitates formation of the grand coalition. However, completely eliminating all asymmetry among states would also eliminate any advantage of policy-coordination. If states
were all identical, Nash equilibria in the game of capital tax competition structure would be efficient for any coalition structure.

Two key modelling choices in this framework are that of a coalition structure rule \( \psi \) and that of a coalitional bargaining rule \( \phi \). As far as coalition structure rules are concerned, it is hard to think of a sensible rule that is outside the class of unanimity-preserving ones. But there are many reasonable alternatives to the Nash bargaining solution \( \phi^* \). How sensitive is the conclusion of Proposition 2 to the choice of the exact bargaining rule? One can show that the grand coalition need not be an equilibrium structure for any bargaining rule with some minimal feature of symmetry as long as the coalition structure rule is the strict unanimity rule \( \psi^* \). We formalize the idea below.

Call a coalition \( T_j \) embedded in the structure \( B \) a coalition of clones if \( T_j \) consists only of states that have identical technology and factor endowments. Call a bargaining rule \( \phi \) weakly symmetric if, for a coalition of clones, \( \phi \) assigns the same payoff to every member. This is a very weak notion of symmetry indeed: it applies only to coalitions of clones. Every single-valued bargaining rule in the literature on cooperative games that we are aware of would satisfy the requirement.

**Proposition 3.** Let \( |N| > 2 \), let the coalition structure rule be the strict unanimity rule \( \psi^* \) and let the coalition bargaining rule \( \phi \) be any weakly symmetric rule. The grand coalition may not be an equilibrium coalition structure.

*Proof.* Consider example (12) again and suppose that the grand coalition is an equilibrium structure under the unanimity rule \( \psi^* \) and some weakly symmetric coalitional bargaining rule \( \phi \). Observe that the values for \( C^*_T(B) \) for each \( B \), reported in (13) above, are independent of both the coalition structure rule \( \psi \) and the coalitional bargaining rule \( \phi \).
For the grand coalition to be an equilibrium structure under the unanimity rule \( \psi^* \), it must be the case that \( \hat{s} = (\hat{S}_1, \hat{S}_2, \hat{S}_3) \) with \( \hat{S}_1 = \hat{S}_2 = \hat{S}_3 = N \) is a CPNE.

A unilateral deviation by any state \( i \) from \( \hat{s} \) to \( S_i = \{i\} \) would lead, under \( \psi^* \), to the singleton coalition structure \( \{\{1\}, \{2\}, \{3\} \} \). Therefore, for \( \hat{s} \) to be even a Nash equilibrium, the coalitional bargaining rule \( \phi \) must assign to each state \( i \) at least what it gets from the singletonal coalition structure. This implies, by (13),

\[
\phi_3(\{N\}) \geq C_3^*(\{\{1\}, \{2\}, \{3\} \}) = 0.823475,
\]
\[
\phi_1(\{N\}) \geq C_i^*(\{\{1\}, \{2\}, \{3\} \}) = 0.07365, \quad i = 1, 2.
\]

A joint deviation by states 1 and 2 from \( \hat{s} \) to \( S_1 = S_2 = \{1, 2\} \) would lead to the structure \( \psi^*(S_1, S_2, \hat{S}_3) = \{\{1, 2\}, \{3\} \} \). For this coalition structure, \( \{1, 2\} \) is a coalition of clones; since \( \phi \) is weakly symmetric, the deviation would yield to each of states 1 and 2

\[
\phi_1(\{\{1, 2\}, \{3\} \}) = \phi_2(\{\{1, 2\}, \{3\} \}) = \frac{C_{12}^*(\{\{1, 2\}, \{3\} \})}{2} = 0.121699.
\]

That joint deviation would be immune to any further unilateral deviation by states 1 and 2 since \( \hat{S}_3 = N \) and

\[
C_i^*(\{\{1\}, \{2\}, \{3\} \}) = 0.07365, \quad i = 1, 2.
\]

Therefore, for \( \hat{s} \) to be a CPNE, such a joint deviation by states 1 and 2 must not be profitable so that either state 1 or state 2 would have to be given at least 0.121699. But then we need

\[
\sum_{i=1}^{3} \phi_i(\{N\}) \geq 0.07365 + 0.121699 + 0.823475 > 1 = C_{123}^*(\{N\}),
\]

a contradiction. This completes the proof of the Proposition. \( \square \)
Note that for the example used in the proof of the Proposition, weak symmetry of the bargaining rule imposes symmetric payoffs for the clones, states 1 and 2, only in the coalition \( \{1, 2\} \), the coalition of clones, not necessarily as members of any other coalition. The proof demonstrates the impossibility of sharing the aggregate consumption of the grand coalition, preserving this minimal symmetry, that would sustain global free trade as an equilibrium outcome under the strict unanimity rule.

Figure 1 here.

Figure 1 illustrates the proof of proposition 3. The payoff frontier for the grand coalition in example (12) is the surface of the unit simplex. The figure shows the portion of the simplex in which \( \phi_3 \geq 0.6 \). For the grand coalition to be an equilibrium structure, the payoffs for the grand coalition must dominate those for the structure \( \{\{1\}, \{2\}, \{3\}\} \). Further, as we argued, to prevent a joint deviation by 1 and 2, leading to \( \{\{1, 2\}, \{3\}\} \), one of those states must receive at least 0.121699. The triangle ABC in the figure is defined by feasibility and state 1 receiving 0.121699. The figure shows that the set of payoffs with \( \phi_1 \geq 0.121699 \) does not intersect the simplex that Pareto dominates the payoffs in \( \{\{1\}, \{2\}, \{3\}\} \). Given the symmetry between states 1 and 2, this means that there is no point in the grand coalition’s payoff frontier from which profitable deviations can be prevented.

The intuition why, once we move beyond a setting with only two states, the grand coalition is no longer necessarily an equilibrium structure is fairly simple. True that whatever the number of states, two or more, the grand coalition always captures all efficiency gains. In other words, corresponding to any assignment of payoffs in any given alternative coalition structure, Pareto improvements can always be found along the payoff frontier of the grand coalition structure. With two states, this is enough to guarantee that each state can improve its payoff over the singleton structure—the
only alternative—by forming a coalition. But when the number of states exceeds two, there are many alternative coalition structures: for example, with three states, there are four other structures. Therefore, the fact that the grand coalition’s payoff frontier dominates that under every other structure, considered separately, is no guarantee that it dominates the alternatives simultaneously.

The lesson we draw from Propositions 2 and 3 is that the mere existence of potential gains from policy coordination, even in the presence of possibilities for costless coalition formation and commitment to transfer schemes to split the gains, does not justify a prediction for the formation of the grand coalition and global free trade.

VII. Discussion

One objective of this paper was to present an integrated and flexible framework for constructing an equilibrium model of an emergent coalition structure that can be adapted to different contexts. We have tried to maintain a modular structure consisting of conceptually distinct components: a model of coalition formation; a model of interaction among coalitions that determines the aggregate payoff for each coalition; and a model of bargaining over that aggregate payoff among members of each coalition. We hope that the modular structure will be useful in facilitating future refinements of each component. Some aspects of our modelling choices in each component especially deserve comment.

In the models of von Neumann and Morgenstern (1944) and Hart and Kurz (1983) that we adopt, a coalition comes about as an outcome of players strategically choosing coalition partners but coalition formation is analyzed as a simultaneous-move game rather than as a dynamic game in which players can make various proposals and counter-proposals. To the extent that any process of coalition formation evolves over
time through rounds of negotiations, the static representation is surely a limitation. But any extensive form one writes down as a description of the protocol of dynamic negotiation is bound to be somewhat arbitrary. And, as is well-known, equilibria in dynamic games are often extremely sensitive to the precise protocols. Moreover, being an equilibrium structure for this static game of coalition formation would seem to be a necessary condition for being an “equilibrium” in any dynamic version of the game. And, our main result is that the grand coalition may not be an equilibrium structure even in the static game. That said, exploring dynamic approaches to modelling coalition formation appears to us to be a highly worthwhile exercise.\textsuperscript{13}

Within the limitation of a static model, the coalition structure rule allows us to distill our intuition about the “underlying dynamics” of a situation in a “reduced form”. This can be an advantage in applications to particular real-world scenarios. For example, for some of the regional trade negotiations currently under way, it is frequently suggested that membership by one or two key players is pivotal to a coalition coming together. Formulating coalition structure rules to reflect such “realities” is quite straightforward; writing down an extensive form appropriate for these negotiations is not. To take a different example, consider the trilateral initiative toward the NAFTA among Canada, Mexico and the USA. A bilateral Free Trade Agreement (FTA) between Canada and the USA was already in place. It seems safe to assume that, during the trilateral negotiations, the players had the common conjecture that the FTA would remain in force even if Mexico opted to not join the initiative; thus, the similarity rule is indicated. Further, for some purposes, a mild regularity condition on the class of coalition structure rules might be enough to settle a question at hand. For Proposition 2 we merely required that the class of admissible

\textsuperscript{13}See Bloch (1992) for a recent paper with a dynamic model of coalition formation.
coalition structure rules be unanimity-preserving.

One feature of the framework in the paper is the distinction between two levels of interaction among the states. Within each coalition, we assume that agreements are binding so that interaction is cooperative. Across coalitions, we suppose that commitments cannot be enforced so that interaction is noncooperative. The dichotomy seems natural since, at the outset, the states are free to form coalitions and enter into binding agreements to capture gains from cooperation. In particular, they may form the grand coalition and reach an efficient outcome. Thus, the boundaries of cooperative behaviour are, in a sense, endogenous.

That there is only a single policy issue in the model does restrict the levels of interaction we can accommodate. In practice, of course, policy issues are invariably multi-dimensional and the scope of interaction is typically multi-layered. There are some issues on which binding agreements are easily enforced, facilitating cooperation up to the level of the grand coalition; some for which accidents of history and geography make commitments practical only among a particular subset of states; others for which the evident lack of any credible enforcement mechanism means the nature of any interaction is fundamentally noncooperative. Accordingly, we often observe a group of states cooperating on some policies but not on others. For instance, the US coordinates some trade policies with a large number of countries under the auspices of the WTO; coordinates a greater range of trade policies, backed by clearer enforcement schemes, with Canada and Mexico under the NAFTA; coordinates aspects of broad macroeconomic policies with the G7 countries; and does not coordinate some tax and social policies even within the country. Capturing the richness of these layers of interaction clearly calls for a model in which the policy space is multi-dimensional and coalitions may overlap.
Turning now to the payoff division with members of each coalition, our analysis is based on solution concepts in cooperative game theory rather than to models of strategic sequential bargaining. Again, as with coalition formation, we recognize that such bargaining processes are inherently dynamic in nature. Still we feel that the recent lessons of sequential bargaining involving more than two players—in brief, the outcome is highly indeterminate—justify our modelling strategy.\textsuperscript{14}

Choosing the Nash bargaining solution as the intra-coalitional bargaining rule, on the other hand, although common in applied work, is admittedly unsatisfactory for games with more than two players. Possibilities of partial cooperation, in between the extremes of full cooperation (among all members within any given coalition) and some exogenously given disagreement point, are not entertained at all. Accordingly, any analysis of intermediate subcoalitions is lacking. From a theoretical perspective, the Shapley value or the nucleolus, both non-empty and single-valued, would be more satisfactory bargaining rules as they explicitly take into account possibilities of cooperation among all subcoalitions.

Adapting standard coalitional analysis, however, poses a problem. The primitive for a game in coalitional form is a \textit{characteristic function} which summarizes (for transferable utility games) the \textit{worth}—aggregate consumption, in our parlance—of every coalition. In the standard theory, the worth of a coalition is independent of

\textsuperscript{14}An innovative recent paper which analyzes multilateral negotiation toward coordination of trade policies using a model of strategic sequential bargaining is Ludema (1994). In Ludema, three countries are involved in bargaining sequentially over gains from trade liberalization. A novel feature is that, during the course of multilateral negotiations, countries may establish bilateral links: a free trade area or a customs union. The potential bilateral agreements therefore act as outside options in the multilateral bargaining process. However, it is assumed that negotiations to complete any remaining links continue even after an outside option is exercised. Ludema’s focus is on how the option of bilateral links and their configuration affect the distribution of payoffs in the final multilateral agreement, rather than on developing a theory of equilibrium coalition structure. In particular, his bargaining model immediately implies that any subgame perfect equilibrium outcome must involve an efficient trilateral agreement.
the structure of complementary coalitions. This is, in general, not true of aggregate consumption for this model: a glance at (13) shows that, for the numerical example in (12),

\[ C^*(\{1\}, \{2\}, \{3\}) = 0.073605 \quad \text{but} \quad C^*(\{1\}, \{2, 3\}) = 0.079229. \]

Pursuing appropriate modifications to the Shapley value and the nucleous that would be suitable for such environments seems worthwhile to us.\(^{15}\)

We are hopeful that our modular framework will also facilitate the study of coalition formation and coalitional bargaining in settings other than capital tax competition. Examples abound of situations in which there are gains to forming coalitions and coordinating strategies. The three-stage modular structure allows us to easily replace the capital tax component and substitute a model appropriate for the underlying example.\(^{16}\)

\(^{15}\)This feature also distinguishes our work from standard club theory and the local public goods literature as in those literatures the worth of a coalition is independent of the structure of complementary coalitions. It is not a feature of club theory by the assumption that public goods are excludable and it is not a feature of the local public goods literature by the assumption that public goods are local. See Suzanne Scotchmer (1994) for a good discussion of these literatures.

\(^{16}\)As an illustration, consider the model of monetary union in Casella (1992). A key example she discusses involves three identical states in which coordination of policies by two federating states allows the third state to benefit by free riding. In particular, the example has the feature that, in any coalition structure of the form \( \{i, j\}, \{k\} \), state \( k \) receives a payoff greater than a third of the grand coalition’s aggregate. With these data it is now easy to show that the grand coalition cannot be an equilibrium structure under the similarity coalition structure rule. The grand coalition, however, may be an equilibrium coalition structure under the strict unanimity rule.
Appendix

Using the implicit function theorem on the system of equations (1) and (2) with manipulation yields

\[
\frac{\partial K_i}{\partial t_i} = \frac{\sum_{i \neq i}^{n} \frac{1}{F_i^{KK}}}{F_i^{KK} \sum_{i=1}^{n} \frac{1}{F_i^{KK}}} < 0, \quad i = 1, \ldots, n, \tag{A.1}
\]

\[
\frac{\partial \rho}{\partial t_i} = -\frac{1}{F_i^{KK} \sum_{i=1}^{n} \frac{1}{F_i^{KK}}} < 0, \quad i = 1, \ldots, n, \tag{A.2}
\]

\[
\frac{\partial K_h}{\partial t_i} = -\frac{1}{F_h^{KK} \sum_{i=1}^{n} \frac{1}{F_i^{KK}}} > 0, \quad h \neq i. \tag{A.3}
\]

Note that summing the right-hand-side of (A.3) over all \( h \neq i \) yields the negative of the right-hand-side of (A.1), or the capital flight out of \( i \) is absorbed by the other states.

Setting \( \partial C_{T_i}(B)/\partial t_i = 0 \), using the definition of \( C_{T_i}(B) \), we get

\[
\sum_{h \in T_j} \left( t_h \frac{\partial K_h}{\partial t_i} - I_h \frac{\partial \rho}{\partial t_i} \right) = 0 \quad \forall i \in T_j
\]

Using (A.1), (A.2) and (A.3) with cancellation yields

\[
\sum_{h \in T_j} \frac{t_h}{F_h^{KK}} - t_i \sum_{h=1}^{n} \frac{1}{F_h^{KK}} = I_{T_j} \quad \forall i \in T_j,
\]

where \( I_{T_j} \) denotes the net capital imports into coalition \( T_j \). Adding and subtracting \( t_i/F_i^{KK} \) we obtain

\[
\sum_{h \in T_j} \frac{t_h}{F_h^{KK}} - t_i \sum_{h=1}^{n} \frac{1}{F_h^{KK}} = I_{T_j} \quad \forall i \in T_j
\]

Solving for \( t_i \) yields (10).
In deriving (10) we assumed that the tax rates to be levied at a Nash equilibrium are dictated by the federal government of the coalition. However, in this simple model, it turns out that any state in coalition $T_j$ would choose to set its capital tax rate, $t_i$ equal to $t_{T_j}$ were it free to do so. To demonstrate this point, we begin by noting that each state knows that at the third stage its final consumption, $\phi^*_i(B)$, will be positively related to the aggregate consumption of its coalition, $C^*_{T_j}(B)$, but also positively or negatively related to the aggregate consumption of potential coalitions which are subsets of $T_j$ (see equation (9)). At the second stage, however, the coalition structure is given. It follows that from the point of view of state $i \in T_j$ maximizing $C^*_{T_j}(B)$ maximizes $\phi^*_i(B)$. Each state $i$ maximizes $C^*_{T_j}(B)$ by choosing $t_i$ taking $t_h$ $h \neq i$ as given. But this yields the same set of first-order conditions as above and thus (10) holds.
References


Ludema, Rodney D. "On the Value of Preferential Trade Agreements in Multilateral
Trade Negotiations." mimeo, University of Western Ontario, 1994.


Table 1: The payoffs for each state in each coalition structure

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</tr>
</thead>
<tbody>
<tr>
<td>(\phi_1^*)</td>
<td>0.073605</td>
<td>0.121699</td>
<td>0.077133</td>
<td>0.079229</td>
<td>0.083377</td>
</tr>
<tr>
<td>(\phi_2^*)</td>
<td>0.073605</td>
<td>0.121699</td>
<td>0.079229</td>
<td>0.077133</td>
<td>0.083377</td>
</tr>
<tr>
<td>(\phi_3^*)</td>
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<td>0.683545</td>
<td>0.827003</td>
<td>0.827003</td>
<td>0.833246</td>
</tr>
</tbody>
</table>
Proposition

Figure 1: Illustration of

For $\mathcal{F} \geq 0.6$ Payoff Frontier