

# Implicit Collusion in Non-Exclusive Contracting under Adverse Selection

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## Abstract

This paper studies how implicit collusion may take place through simple non-exclusive contracting under adverse selection when multiple buyers (e.g., entrepreneurs with risky projects) non-exclusively contract with multiple firms (e.g., banks). It shows that any price schedule can be supported as equilibrium terms of trade in the market if each firm's expected profit is no less than its reservation profit. Firms sustain collusive outcomes through the triggering trading mechanism in which they change their terms of trade contingent only on buyers' reports on the lowest average price that the deviating firm's trading mechanism would induce.

## 1 Introduction

Trading in decentralized markets often take place when one trader has private information that features common values in the sense that it affects not only his payoffs but also the payoff of the trader whom he trades with: For example, a car owner know the quality of his car but buyers do not observe it. Akerlof (1970) showed how this type of the market for lemons is operated in a decentralized economy. The analysis is based on the competitive market equilibrium. It admits multiple equilibria so that aggregate equilibrium allocations differ across equilibria but every equilibrium has the same qualitative properties: Transaction price correctly reflects the average quality of

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the good traded in the market and only bad-quality goods are traded in every equilibrium.

Rothchild and Stiglitz (1976) show how firms strategically compete in the competitive market for lemons. When an insurance company is able to use a menu of contracts to screen the individual's inherent risks, the adverse selection problem can be partially mitigated in the sense that both types of the individual will buy an insurance contract. In equilibrium, insurance companies have all zero profits. Equilibrium may not exist at all especially when a large fraction of individuals have low risks.

However, it is not clear whether the non-existence problem is something inherent in the market for lemons or it is due to the restriction of the exclusive trading imposed in their model; an individual can buy insurance from only one insurance company. In fact, trading in a decentralized market is frequently non-exclusive by nature. For example, an entrepreneur may borrow money from multiple banks to finance his risky project. A buyer who faces the underlying asset risks associated with interest rate, credit, or foreign exchange may buy contingent claims from multiple sellers to diversify those risks in the risk transfer markets.

Attar, Mariotti, and Salanié (2011) provide a noble strategic foundation of the lemon's problem (Akerlof 1970) in non-exclusive trading where a single buyer buys the good from multiple sellers. They showed that equilibrium in fact exists under mild conditions in non-exclusive trading. Their results are consistent with Akerlof in the sense that the transaction price correctly reflects the average quality of the good traded in the market. However, aggregate equilibrium allocations are shown to be unique in their model.

Non-exclusive trading in fact happens on both sides of the market; buyers with the underlying risks buy contingent claims from multiple sellers and sellers sell contingent claims to multiple buyers as well in the risk transfer markets; entrepreneurs borrow money from multiple banks and banks lend money to multiple entrepreneurs as well in loan contracting. When we model non-exclusive trading explicitly on both sides of the market, it raises a new scope of negotiation for parties who offer contracting schemes. For example, when a bank negotiates a pair of principal and repayment with a borrower, it can make its offer based not only on communication with that borrower but also on communication with all the other borrowers.

Generally, non-exclusive contracting with multiple buyers (e.g., entrepreneurs in loan contracting) is generally a complex process for firms (e.g., banks in loan contracting, sellers) because buyers can also contract with competing firms. In this contracting environment, buyers may well communicate with firms at the contracting stage because firms can ask buyers about competing firms'

terms of trade (e.g. principal and repayment pairs in loan contracting). Importantly, when multiple buyers communicate with firms, firms can compare what buyers are telling. This may make it easier for firms to acquire the true information on competing firms' terms of trade from buyers' reports on competing firms' terms of trade. Subsequently, firms may want to offer their negotiation schemes (formally, trading mechanisms) in which their terms of trade depend on buyers' reports on competing firms' terms of trade. In this way, firms can punish a deviating firm by changing their terms of trade upon buyers' reports on the deviating firm's terms of trade and hence they may sustain many collusive outcomes.

The idea of collusion through complex negotiation schemes motivates the literature on competing mechanism design in which, for example, multiple firms compete in designing their trading mechanisms (Epstein and Peters 1997, Yamashita 2010). However, the languages that are required for buyers to use in the negotiation schemes are quite complex.<sup>1</sup> Furthermore, in order to punish the deviating firm, buyers play the (worst) continuation equilibrium for the deviator upon his deviation to an arbitrary complex negotiation schemes while the other firms offers what they are supposed to offer. However, the literature on competing mechanism design does not show how to derive the continuation equilibrium that punishes the deviator upon his deviation to an arbitrary negotiation schemes because it focuses on a general methodology. For these reasons, very few economic applications have been developed despite of its huge potential on applications.

Given the prevailing examples of non-exclusive trading under adverse selection, it is quite important to develop a model that provides tractable negotiation schemes for various collusive outcomes among firms. In this context, the simplicity of a buyer's communication seems important to understand implicit collusion in the applications of non-exclusive trading problems. The purpose of this paper is two folds: First, it aims to develop a simple equilibrium mechanism that can minimize the buyer's communication burden, for a better understanding of implicit collusion in non-exclusive contracting under adverse selection such as investment financing, insurance, and various other trading problems. Second, it completely characterizes the continuation equilibrium that punishes the deviating firm upon its deviation to any arbitrary trading mechanism.

Consider a market for a good where each privately-informed buyer can buy

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<sup>1</sup>The buyer needs to send a message that is an infinite sequence of real numbers (Epstein and Peters 1999) or need to recommend to the firm an entire mapping (i.e., direct mechanism) from buyers' types to the firm's actions that the firm should implement (Yamashita 2010, Peters and Troncoso Valverde 2012).

from any number of firms and each firm can also sell its product to any number of privately-informed buyers. Firms can freely offer any arbitrary trading mechanism that make quantity and monetary payment pairs across buyers contingent on their messages. The market terms of trade can be characterized by a price schedule that specify monetary payment from the buyer as a function of the quantity that the buyer buys. The key result of the paper is to show how to construct an equilibrium trading mechanism for firms, given their implicit agreement on a price schedule, in a way that no firm gains by deviating to any arbitrary complex trading mechanism. Then, we show that any price schedule can be supported as equilibrium terms of trade in the market as long as it ensures that each firm receives no less profit than its reservation profit.

This paper proposes the triggering trading mechanism with which firms can maintain their implicit agreement on a price schedule, say  $\tilde{y}$ . A triggering trading mechanism asks each buyer to report, along with the quantity that he wants to buy from the firm, whether there is a deviating firm and, if so, what would be the deviating firm's lowest average price that he believes he would face if he was the only one who bought from the deviating firm. When buyers are anonymous so that the trading mechanism is anonymous, each buyer has the same belief on the lowest average price that the deviating firm's trading mechanism would induce when he would be the only one who participated in the deviating firm's trading mechanism. As shown later, this approach is easily extended to the case in which sellers offer different price schedules to ex-ante heterogeneous buyers.

The triggering trading mechanism has the following structure. When two or more buyers participate in a firm's triggering trading mechanism, and more than half of their reports on the deviating firm's lowest average price are all  $p$ , then the firm offers a linear price schedule such that its unit price matches the minimum between  $p$  and the lowest average price of  $\tilde{y}$ , which is a price schedule firms implicitly agree on. In all other cases, the firm continues to offer  $\tilde{y}$ .

Let us now characterize the continuation equilibrium that punishes the deviating firm upon its deviation to any arbitrary trading mechanism. When a firm deviates to an arbitrary trading mechanism, each buyer reports his true belief  $p$  to non-deviating firms. Then, each non-deviating firm's price schedule is the linear price schedule in which the unit price matches the minimum between  $p$  and the lowest average price of  $\tilde{y}$ . When there are three or more buyers, one buyer cannot unilaterally change the non-deviating firm's price schedule given the other buyers' truthful reports,  $p$ .

When there are only two buyers and their reports on the deviating firm's lowest average price are different, a non-deviating firm does not know which

buyer is telling a lie. Note that in this case, the triggering trading mechanism continues to offer  $\tilde{y}$  for both of them. This prevents a buyer's lie in the two-buyer case because the linear price schedule with the unit price equal to the minimum between  $p$  and the lowest average price of  $\tilde{y}$  is always better for both buyers than the original price schedule  $\tilde{y}$ .

Therefore, whether there are two buyers or more, each buyer truthfully reports  $p$  to each non-deviating firm given that the other buyers do the same. Furthermore, it is also optimal for each buyer to buy only from non-deviating firms because the linear price described above always provides better terms of trade for the buyer than the deviating firm's trading mechanism does when he is only one who participates in the deviating firm's trading mechanisms. Consequently, a deviating firm ends up with its reservation profit upon any deviation to any arbitrary mechanism because no agents buys from the deviating firm in truthful continuation equilibrium.

When no firm deviates, each buyer truthfully reports each firm, along with the quantity that he wants to buy from each firm, that no firm has deviated and then each firm continues to offer  $\tilde{y}$ . Because all buyers report that no firm has deviated, each firm also continues to offer  $\tilde{y}$  upon any buyer's unilateral deviation to an alternative message and hence no buyer has an incentive to tell a lie. As long as a price schedule ensure that each firm receives no less profit than its reservation profit, no firm has an incentive to deviate to any arbitrary trading mechanism because it only receives its reservation payoff upon deviation to any trading mechanism.

The triggering trading mechanism features convenience in a large class of applications. Because each buyer's message is simply two numbers (the deviating firm's lowest average price and the quantity that the buyer wants to buy), it is simple and independent of the number of buyers. Finally, it also works for any multiple number of buyers, including the case of two buyers, and the set of equilibrium payoffs is defined in terms of each firm's reservation profit which is independent of trading mechanisms.

Akerlof's analysis induces zero-profit equilibrium in the general equilibrium framework while it admits multiplicity of equilibrium. Attar, Mariotti, and Salanié (2011) extends Akerlof's analysis to the non-exclusive trading environment where a privately-informed buyer buys a good from multiple firms compete: They show that firms receive zero profit in equilibrium because the equilibrium price equals the average quality of the good traded in the market. Those equilibrium analyses assume that a firm may not utilize what it learns from communication with other buyers when it negotiates terms of trade with a buyer. If it does, it can compare what the buyer who is currently negotiating with it says about the other competing firms' terms of trade with what the

other buyers say. This leads the buyer to reveal his true information on the competing firms' terms of trade. Of course, the negotiating process observed in practice may be quite complex but if it shows this type of negotiation features, then firms can in fact maintain not only linear price schedules but also various price schedules that lead to collusive outcomes. In this case, the average price derived from the prevailing price schedule may not necessarily reflect the quality of the good traded in the market. This has very important implications. It suggests that a good (e.g., financial contract in the risk transfer market) can be overpriced in a competitive market even with fully rational traders and without any explicit and public collusive agreement among firms.

Section 2 briefly introduces the literature on trading problems under adverse selection and competing mechanism design. Section 3 sets up the model. Section 4 establishes the main result on how firms maintain implicit collusion. Section 4 provides in-depth discussion on the link and differences between our results and those in the literature. Section 5 concludes the paper.

## 2 Literature Review

The adverse selection problems in the market for lemons are well known from Akerlof (1970) or Rothchild and Stiglitz (1976). While Rothchild and Stiglitz showed that the firm's screening menu can mitigate the adverse selection problems, it also raises the market failure in the sense that equilibrium may not exist at all. Jaynes (1978) and Hellwig (1988, 2011) showed that when insurance firms directly disclose and share information on who accepts the insurance contract, the non-existence problem of equilibrium under exclusive contracting can be resolved.

To study adverse selection problems in a more general framework, non-exclusive trading is studied in the environment where a single buyer (e.g., entrepreneur in loan contracting) can buy the good from multiple firms. Attar, Mariotti, and Salanié (2011) consider non-exclusive trading problems in which both firms and the buyer have linear preferences over quantity and price and the buyer's type is continuous. They showed that equilibrium always exists under mild conditions.<sup>2</sup> In equilibrium, non-trivial pooling leads to cross-

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<sup>2</sup>Prat and Rustichini (2003) extend non-exclusive trading to bilateral contracting in which multiple principals negotiate terms of trades with multiple agents independently. However, agents have no private information in Prat and Rustichini's model. Attar, Mariotti, and Salanie (2011) show how to extend their results under adverse selection to bilateral contracting with menus of price-quantity pairs (Han (2006) shows why principals can rely on menus instead of complex mechanisms in bilateral contracting).

subsidization across different types. They show that equilibrium aggregation allocation is unique and each firm receives zero profit given its linear price schedule. Because the buyer can purchase the good from multiple firms, competing firms' competitive price schedules prevent any firm from selling more goods to the buyer without making loss. In particular, linear price schedules in equilibrium prevent cream-skimming strategies. The unit price embedded in the linear price schedule correctly reflects the average quality of the good traded in the market.

Attar, Mariotti, and Salanié (2012) study non-exclusive trading problems when the buyer has strictly convex preferences with the single crossing property but firms have linear preferences. Compared to their early paper (2011), the model in Attar, Mariotti, and Salanié (2012) is also different in the following aspects: (a) the buyer's type is discrete and admits only two possible values and (b) no quantity constraint that the buyer can trade. They showed that the existence of equilibrium is no longer guaranteed. Pooling occurs only when there is no trading at all in equilibrium. In equilibrium with trading, there is no cross-subsidization because only one type of the buyer is served. Aggregation equilibrium allocation is unique and each firm gets zero profits. Ales and Mazeiro (2012) derives similar results with three types for the buyer.

While Attar, Mariotti, and Salanié (2011, 2012) and Ales and Mazeiro (2012) derive Bertrand-type equilibrium outcomes, Biais, Martimort, and Rochet (2000) derive qualitatively different results; Assumptions on preferences are the same as those in Attar, Mariotti, and Salanie (2012) but Biais, Martimort, and Rochet (2000) allows for continuous buyer types in the market where market makers compete in price schedules to supply liquidity to a single agent who is privately informed about the value of the asset and his hedging needs. In contrast to Attar, Mariotti, and Salanié (2012), the continuity of the type induces Cournot-type equilibrium outcomes in Biais, Martimort, and Rochet (2000): there exists a unique equilibrium in convex price schedules; each market maker makes positive expected profits but these profits go away as the number of market makers increases.

As discussed above, equilibrium allocation properties in non-exclusive trading with a single buyer and multiple firms depend on primitives of the model such as the buyer's risk attitude (risk neutral vs. risk averse) and the buyer's private information structure (discrete vs. continuous). However, it seems difficult to support overpricing of financial contracts and/or getting expected profits above the competitive level in a competitive market. The literature on competing mechanism design theoretically suggests how various outcomes can be supported when principals (e.g., firms) may design mechanisms that ask agents (e.g., buyers) to report the market information that they have such as

what they know about the other principals' mechanisms and so on. Epstein and Peters (1999) construct a very rich language that agents can use in describing the market information when they communicate with firms. However, their language is quite complex to apply.<sup>3</sup>

Yamashita (2010) shows that firms can sustain various outcomes if each firm offers the recommendation mechanism that asks each agent to report his type and the direct mechanism the firm should choose. When all agents report the same direct mechanism, the firm chooses that direct mechanism, which then determines the firm's decision according to agents' type reports. His approach tells us how one can view firms' implicit collusion via their commitment to the recommendation mechanisms.

The recommendation mechanism needs at least three or more agents for their truthful reports but it provides a perfectly nice way of understanding implicit collusion in general. Each agent's message in the recommendation mechanism is simpler than the message in the universal language (Epstein and Peters 1999). However, the message in the recommendation mechanism is still complex and in particular it becomes increasingly complicated as the number of agents increases. The reason is that each agent must report the entire mapping of a direct mechanism that specifies an action for every possible profile of all agents' types and hence each agent's burden of communication exponentially increases in the number of agents.

Extending Yamashita's approach, Peters and Trancoso Valverde (2012) show that any incentive compatible and individually rational allocations that the central mechanism designer offers can be supported in competing mechanism games.<sup>4</sup> However, their model differs from the usual framework in that it allows everyone to offer mechanisms to everyone else, erasing the distinction between principals and agents. It implies that all players observe when one player deviates to change his mechanism. Results in Peters and Trancoso Valverde require that all players send and receive messages because of

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<sup>3</sup>In common agency (multiple firms and a single agent), Pavan and Calzolari (2009, 2010) propose a tractable class of extended direct mechanisms that can be used in deriving an equilibrium relative to any complex mechanisms or equivalently menus (Peters 2001 and Martimort and Stole 2002). They show that a firm can ask the agent about his choice of payoff-relevant alternatives from all the other firms, along with his type. The agent's communication is simpler than the communication with the universal language (Epstein and Peters 1999) or the communication in the recommendation mechanism (Yamashita 2010). However, it is not obvious how to extend Pavan and Calzolari's approach to multiple agency (i.e., multiple firms and multiple agents).

<sup>4</sup>Yamashita does not identify equilibrium allocations because, in his approach, equilibrium allocations are specified by the firm's minmax value relative to the set of all complex mechanisms but it is not feasible to specify the exact set of all complex mechanisms.

two rounds of communication — each player receives messages (e.g., recommended direct mechanisms and types) from all participating players in the first round and then relays the type reports (from participating players) to everyone else in the second round to confirm whether each player in fact reported the same type to everyone else. This type of communication may not be feasible because, for example, it is not legal for competing firms to actively communicate one another for collusion.

### 3 Model

Non-exclusive trading under adverse selection is frequently observed in practice. For example, consider **risk transfer markets** where buyers face underlying asset risks but their preferences may differ over the desire to shed that risk: Buyer  $i$  holds an asset that can take one of the two-state contingent values: it returns  $r > 0$  with probability  $\omega_i$  (good state) but nothing with probability  $1 - \omega_i$  (bad state). The quality of the underlying asset is characterized by  $\omega_i$ , which is often buyer  $i$ 's private information. Buyer  $i$  can purchase contingent claims that pay in the bad state from one or several sellers. Let  $a_i^j \subset A$  be the amount of a contingent claim that buyer  $i$  purchases from seller  $j$ , where  $A := [0, \bar{a}]$  denotes the set of all feasible amounts of the contingent claim that each seller can sell to each buyer. Let  $t_i^j \geq 0$  be the price that buyer  $i$  pays to seller  $j$  for the contingent claim  $a_i^j$ . Let  $U(\cdot)$  be each buyer's Bernoulli utility function for money. Since buyer  $i$  can buy contingent claims from any number of sellers, his expected utility is

$$\omega_i U(r - t_i) + (1 - \omega_i) U(a_i - t_i),$$

where  $(a_i, t_i) = (\sum_{k=1}^J a_i^k, \sum_{k=1}^J t_i^k)$ . Seller  $j$  can sell his contingent claim to any number of buyers. When the seller is risk neutral, his expected profit associated with the vector of contingent claims  $[a_1^j, \dots, a_I^j]$  that he sells and the vector of prices  $[t_1^j, \dots, t_I^j]$  is

$$\sum_{k=1}^I t_k^j - \sum_{k=1}^I (1 - \omega_k) a_k^j$$

at  $\omega = [\omega_1, \dots, \omega_I]$ .

Another example of non-exclusive trading under adverse selection can be found in **loan contracting**. Entrepreneur  $i$  has a risky investment project. It generates profit  $f(z_i)$  when the amount of money invested in the project is  $z_i$ . Let  $Z := [0, \bar{z}]$  be the set of feasible amounts of money that each entrepreneur

can borrow from each lender. Denote by  $z_i^j \in Z$  the amount of money that entrepreneur  $i$  can borrow from lender  $j$ . Let  $p_i^j \geq 0$  be the amount of money that the entrepreneur agrees to pay back when the project turns out to be successful. Let  $\omega_i$  be the probability of success. As entrepreneur  $i$  can borrow money from multiple lenders, his expected payoff is

$$\omega_i[f(z_i) - p_i],$$

where  $(z_i, p_i) = (\sum_{k=1}^J z_i^k, \sum_{k=1}^J p_i^k)$ . Let  $\rho$  be the risk-free (gross) interest rate. As lender  $j$  can lend money to multiple entrepreneurs, his expected profit associated with  $[z_1^j, \dots, z_I^j]$  and  $[p_1^j, \dots, p_I^j]$  is

$$\sum_{k=1}^I \omega_k p_k^j - \rho \sum_{k=1}^I z_k^j$$

at  $\omega = [\omega_1, \dots, \omega_I]$ . The quality of entrepreneur  $i$ 's project is characterized by the probability of success and it is often only observable by the entrepreneur.

There are also other examples of non-exclusive trading. Each investor hires multiple investment advisors for **investment advice** and each investment adviser may work for multiple investors. Each buyer may buy **insurance contracts** from more than one insurance companies and each insurance company sells insurance contracts to multiple buyers.

We set up a general model in the context of firms and buyers but it can be applied to any non-exclusive trading problems. There are  $J$  firms (e.g., sellers in the risk transfer markets, lenders in loan contracting) and  $I$  ex-ante identical buyers (e.g., buyers in the risk transfer markets, entrepreneurs in loan contracting) in a market for a perfectly divisible good.

Assume  $J \geq 2$  and  $I \geq 2$ . Each buyer can buy the good from one or more firms. Let  $x_i^j \in X$  denote the quantity of the good that buyer  $i$  buys from firm  $j$ , where  $X := [0, \bar{x}]$  be the set of feasible quantities that each firm can sell to each buyer.<sup>5</sup> Let  $m_i^j \in \mathbb{R}_+$  be the monetary payment from buyer  $i$  to firm  $j$ . Let  $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k)$  be the pair of the total quantity that buyer  $i$  buys from firms and the total monetary payment that he makes to them. Let  $\omega_i$  denote buyer  $i$ 's payoff type, which is assumed to be buyer  $i$ 's own private information. Let  $\Omega$  be the set of all feasible payoff types for each buyer.

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<sup>5</sup>We set up  $X$  to be a closed connected interval for technical simplicity. The upper-bound  $\bar{x}$  can be thought of as the firm's capacity constraint. If there is no capacity constraint, one can set up the value of  $\bar{x}$  sufficiently high so that the upper-bound is never binding in equilibrium.

When buyer  $i$  of type  $\omega_i$  trades the total quantity  $x_i$  at the total payment  $m_i$ , his utility is

$$u(x_i, m_i, \omega_i).$$

We assume that  $u(x_i, m_i, \omega_i)$  is increasing in  $x_i$  and decreasing in  $m_i$ . Each firm  $j$ 's profit associated with  $[x_1^j, \dots, x_I^j]$  and  $[m_1^j, \dots, m_I^j]$  is denoted by

$$v^j(x_1^j, \dots, x_I^j, m_1^j, \dots, m_I^j, \omega)$$

at each  $\omega = [\omega_1, \dots, \omega_I]$ . We assume that  $v^j(x_1^j, \dots, x_I^j, m_1^j, \dots, m_I^j, \omega)$  is decreasing in each  $x_i^j$  and increasing in each  $m_i^j$ . In the examples above, private information that a buyer holds features common value in the sense that it affects both his utility and the firm whom he trades with. Therefore, we incorporate the common value feature to make firm  $j$ 's profit function dependent on buyers' types. It is also important to note that we impose the usual monotonicity property on the utility function and profit function but not the single crossing property nor linearity.

### 3.1 Competition in Trading Mechanisms

As buyers search for a better deal, they may be better informed than firms about what have been offered by firms in the market. Even if a firm may not observe what have been offered by the competing firms, it may ask buyers about what they know about the competing firms' offers in order to make its offer responsive to the competing firms' offers. Buyers may also have incentives to communicate with firms about the competing firms' offers in order to receive better offers.

Abstracting from the reality, we consider contracting in which firms may freely offer buyers any arbitrary trading mechanisms. We assume that firms do not observe trading mechanisms offered by competing firms. An alternative interpretation is that firms do observe competing firms' trading mechanisms but they cannot write binding contracts directly contingent on competing firms' offers that they observe. However, in their trading mechanisms, firms can make their terms of trade for a buyer contingent on all buyers' reports. Messages are private in the sense that the message that buyer  $i$  sends to firm  $j$  are observable only between them. This is consistent with the formulation in Epstein and Peters (1999) and Yamashita (2010).

A firm's trading mechanism determines the quantity and payment pair for each buyer contingent on all buyers' messages. For each firm  $j$ , let  $C$  be the set of messages available for each buyer  $i$ . Because buyers are ex ante anonymous, the firm offers an anonymous trading mechanism. Given firm  $j$ 's

trading mechanism  $\gamma^j : C^I \rightarrow X \times \mathbb{R}_+$ ,  $\gamma^j(c_i^j, c_{-i}^j) \in X \times \mathbb{R}_+$  denotes the quantity and payment pair for each buyer  $i$  when his message is  $c_i^j$  and the other buyers' messages are  $c_{-i}^j$ . For notational simplicity, let  $C$  include the null message  $\emptyset$ . We assume that if a buyer decides not to participate in firm  $j$ 's trading mechanism, it is equivalent to sending the null message  $\emptyset$  to firm  $j$ . Let  $\gamma^j(C, c_{-i}^j)$  denote the set of all quantity and monetary payment pairs that each buyer  $i$  can induce by sending messages in  $C$  when the other buyers' messages are  $c_{-i}^j$ .

Let  $\Gamma^j$  be the set of all feasible trading mechanisms for each firm  $j$ . Let  $\Gamma \equiv \times_{k=1}^J \Gamma^k$ . A competing mechanism game relative to  $\Gamma$  starts when each firm  $j$  simultaneously offers a trading mechanism from  $\Gamma^j$ . After observing a profile of trading mechanisms, each buyer sends messages, one to each firm. Each firm  $j$  decides quantity and monetary payment pairs, one for each buyer, contingent on the messages that it receives from buyers. A trading mechanism can be very complex because the set of messages in a trading mechanism can be quite general in the degree and nature of the communication that it permits regarding what the other firms are doing: It could ask the buyer to report not only about his type but also about the whole set of trading mechanisms offered by the other firms, the terms of trade that the buyer chooses from the other firms, and so on. We adopt the notion of perfect Bayesian equilibrium for the solution concept of the competing mechanism game relative to  $\Gamma$ .

## 4 Implicit Collusion

Now we examine how firms can maintain their implicit collusion on terms of trade. The market terms of trade can be characterized by a price schedule  $\mathbf{y} : X \rightarrow \mathbb{R}_+$ . A price schedule  $\mathbf{y}$  specifies the buyer's non-negative payment to a firm as a function of the quantity that he buys from the firm and it satisfies  $\mathbf{y}(x) = 0$  if  $x = 0$ .

Suppose that firms implicitly agree that they will trade with buyers according to a price schedule  $\tilde{\mathbf{y}}$ . If the buyer can buy the good from each firm according to the price schedule  $\tilde{\mathbf{y}}$ , his payoff maximization problem can be stated as follows: For each  $\omega_i \in \Omega$ ,

$$\max_{(x^1, \dots, x^J) \in X^J} u \left( \sum_{k=1}^J x^k, \sum_{k=1}^J \tilde{\mathbf{y}}(x^k), \omega_i \right) \quad (1)$$

Let  $(\tilde{x}^1(\omega_i), \dots, \tilde{x}^J(\omega_i))$  be a solution to problem (1). Then, the maximum

payoff for buyer  $i$  of type  $\omega_i$  becomes

$$\tilde{U}(\omega_i) \equiv u \left( \sum_{k=1}^J \tilde{x}^k(\omega_i), \sum_{k=1}^J \tilde{\mathbf{y}}(\tilde{x}^k(\omega_i)), \omega_i \right).$$

Let  $u_o(\omega_i) \equiv u(0, 0, \omega_i)$  be the reservation utility for the buyer of type  $\omega_i$ . Because  $\tilde{\mathbf{y}}(x) = 0$  for  $x = 0$ , we can assure that  $\tilde{U}(\omega_i) \geq u_o(\omega_i)$  for all  $\omega_i \in \Omega$ .

Given a price schedule for each buyer, the expected payoff for firm  $j$  can be accordingly expressed as

$$V^j(\tilde{\mathbf{y}}) \equiv \mathbb{E} [v^j(\tilde{x}^j(\omega_1), \dots, \tilde{x}^j(\omega_I), \tilde{\mathbf{y}}(\tilde{x}^j(\omega_1)), \dots, \tilde{\mathbf{y}}(\tilde{x}^j(\omega_I))), \omega)],$$

where  $\mathbb{E}[\cdot]$  is the expectation operator over  $\omega = [\omega_1, \dots, \omega_I]$ . Let  $v_o^j \equiv \mathbb{E}[v^j(0, \dots, 0, \omega)]$  be the reservation profit for firm  $j$  when it does not sell at all.

We now examine how firms can implicitly support any price schedule  $\tilde{\mathbf{y}}$  with  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$ , as their equilibrium terms of trade. To this end, we first construct each firm's equilibrium trading mechanism that prevents any firm's deviation to any complex trading mechanism. We call it a triggering trading mechanism.

For an arbitrary price schedule  $\tilde{\mathbf{y}}$  with  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$ , each firm  $j$ 's **triggering trading mechanism** is denoted by  $\gamma_E^j: E^I \rightarrow X \times \mathbb{R}_+$ . The set of messages available for each buyer  $i$  is  $E \equiv P \times X$ , where  $P = \mathbb{R}_+ \cup \{\eta\}$ . Each buyer  $i$  reports  $(p, x) \in E$ .<sup>6</sup> The message  $x \in X$  is the quantity that the buyer wants to buy from the firm. The message  $p$  has the following meaning. If  $p = \eta$ , then it means either (i) no other firms deviate from the triggering trading mechanisms or (ii) a deviating firm's price schedule for each buyer is  $\tilde{\mathbf{y}}$  and it is independent of the other buyers' messages to the deviating firm. If  $p \in \mathbb{R}_+$ , then it means (a) there exists a deviating firm whose trading mechanism does not induce (ii) and (b)  $p$  is the the deviating firms' lowest average price for the buyer if he was the only buyer who participated in the deviating firm's mechanism.

Suppose that firm  $k$  deviates to a mechanism  $\gamma^k: C^I \rightarrow X \times \mathbb{R}_+$ . When each buyer  $i$  is the only buyer who participates in the deviating firm's mechanism, the deviating principal's lowest average price for the buyer is defined as

$$\inf \left\{ p' \in \mathbb{R}_+ : p' = \frac{m}{x} \text{ for } (x, m) \in \gamma^k(C, \emptyset^{I-1}) \text{ and } x \neq 0 \right\}.$$

For an arbitrary price schedule  $\tilde{\mathbf{y}}$  with  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$ , the triggering trading mechanism  $\tilde{\gamma}_E^j: E^I \rightarrow X \times \mathbb{R}_+$  has the following properties:

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<sup>6</sup>If an agent decides not to buy from a firm, it is assumed to be equivalent to sending  $x = 0$  to the firm. Accordingly the mechanism assigns zero monetary payment for the agent.

- D1.** If the number of participating buyers is two or more and more than half of participating buyers report  $p \in \mathbb{R}_+$ , the firm offers a linear price schedule  $\tau(p)$  such that  $\tau(p)(x) = ax$  for all  $x \in X$ . Each participating buyer  $i$  then pays  $\tau(p)(x) = ax$  for the quantity  $x \in X$  that he submits along with his report on some other firm's lowest average price.
- D2.** In all other cases, the price schedule is  $\tilde{\mathbf{y}}$ . Each buyer  $i$  then pays  $\tilde{\mathbf{y}}(x)$  for the quantity  $x \in [0, 1]$  that he submits along with his report on some other firm's price schedule.

The key to the triggering trading mechanism is to set up  $\tau(p)$  for all  $p \in \mathbb{R}_+$  in a way that it induces buyers not to buy from a deviating firm in truth-telling continuation equilibrium. As shown later, non-deviating firms' triggering trading mechanisms in fact lead to truth-telling continuation equilibrium in which each buyer reports, to each non-deviating firm, the lowest average price  $p$  that he believes he would face from the deviating firm if he was the only one who participated in the deviating firm's trading mechanism.

Suppose that a deviating firm's price schedule is  $p \in \mathbb{R}_+$  for each buyer if he was the only one who bought from the deviating firm. When two or more buyers participate in the non-deviating firm's triggering trading mechanism and more than half of participating buyers report  $p \in \mathbb{R}_+$ , then the triggering trading mechanism assigns the linear price schedule  $\tau(p)(x) = ax$  that satisfies

$$a = \min \left[ p, \inf_{x \in (0, \bar{x}]} \left( \frac{\tilde{\mathbf{y}}(x)}{x} \right) \right]. \quad (2)$$

Note that  $\inf_{x \in (0, \bar{x}]} \left( \frac{\tilde{\mathbf{y}}(x)}{x} \right)$  is the lowest average price based on the price schedule  $\tilde{\mathbf{y}}$ .

Consider an arbitrary  $\tilde{\mathbf{y}}$  that induces  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$ . Our main result shows that when every firm offers the triggering trading mechanism with  $\tau(\cdot)$  that satisfies (2) for any  $p \in \mathbb{R}_+$ , there exists the truth-telling continuation equilibrium in which no firm  $j$  can make more profit than  $V^j(\tilde{\mathbf{y}})$  by deviating to any complex trading mechanism. Therefore, any price schedule  $\tilde{\mathbf{y}}$  with  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$  can be supported as equilibrium terms of trade in the market.

**Theorem 1** *Suppose that each firm offers the triggering trading mechanism associated with a price schedule  $\tilde{\mathbf{y}}$  with  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for all  $j$ . It is the equilibrium mechanism for each firm in perfect Bayesian equilibrium in which the truth-telling continuation equilibrium is characterized as follows:*

1. When no firm deviates or firm  $k$  deviates to a mechanism that induces  $\tilde{\mathbf{y}}$  to each buyer regardless of the other buyers' reports to firm  $k$ , each buyer  $i$  of type  $\omega_i$  sends the message  $(\eta, \tilde{x}^j(\omega_i))$  to each non-deviating firm  $j$  and a message, to firm  $k$ , which leads him to buy  $\tilde{x}^k(\omega_i)$  at  $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$  from firm  $k$ .
2. When firm  $k$  deviates to any other mechanism, each buyer  $i$  of type  $\omega_i$  buys  $\hat{x}(\omega_i)$  only from every non-deviating firm by reporting  $(p, \hat{x}(\omega_i))$ , where  $p$  is each buyer's belief on the lowest average price that the deviating firm's mechanism would induce if only one buyer participated in its mechanism and  $\hat{x}(\omega_i)$  satisfies

$$\hat{x}(\omega_i) \in \arg \max_{x \in X} ((J-1)x, (J-1)\tau(p)(x), \omega_i)$$

**Proof.** Choose an arbitrary price schedule  $\tilde{\mathbf{y}}$  that induces  $V^j(\tilde{\mathbf{y}}) \geq v_o^j$  for each firm  $j$  based on the solution  $(\tilde{x}^1(\omega_i), \dots, \tilde{x}^j(\omega_i))$  to problem (1). Each firm offers the triggering trading mechanism associated with the price schedule  $\tilde{\mathbf{y}}$ . We will show that the triggering trading mechanism is the equilibrium trading mechanism for each firm in perfect Bayesian equilibrium in which buyers truthfully communicate with non-deviating firms on their beliefs on the lowest average price that a deviating firm's trading mechanism would induce no matter how complex the deviating firm's trading mechanism is. First of all, consider the truth-telling continuation equilibrium on the equilibrium path

**(a) On the equilibrium path:** When no firm deviates from its triggering trading mechanism, each buyer  $i$  participates in all firms' triggering trading mechanisms by sending the message  $(\eta, \tilde{x}^j(\omega_i))$  to each firm  $j$ . Suppose that a buyer considers a deviation from the report  $\eta$  when he communicates with a firm. Because of condition (D2), a buyer cannot unilaterally change a firm's price schedule away from  $\tilde{\mathbf{y}}$  with any other report in  $P$  given that all the other buyers send  $\eta$  to the firm. Therefore, it is incentive compatible for each buyer to send  $\eta$  to each firm when the other buyers also send  $\eta$  to each firm. Because each firm's price schedule becomes  $\tilde{\mathbf{y}}$ , it is in fact optimal for each buyer  $i$  of type  $\omega_i$  to participate in each firm  $j$ 's triggering trading mechanisms by sending  $\tilde{x}^j(\omega_i)$  along with  $\eta$ .

**(b) Off the equilibrium path:** Now we consider firm  $k$ 's deviation to any complex trading mechanism. There are two types of deviation.

**(b-1)** Suppose that firm  $k$  deviates to a trading mechanism  $\gamma^k: C^I \rightarrow X \times \mathbb{R}_+$  such that (i) for all  $c_i^k \in C$  and all  $c_{-i}^k, \hat{c}_{-i}^k \in C^{I-1}$ ,

$$\gamma^k(c_i^k, c_{-i}^k) = \gamma^k(c_i^k, \hat{c}_{-i}^k) \quad (3)$$

and (ii) for each  $x \in X$ ,

$$\min\{m \in \mathbb{R}_+ : (x, m) \in \gamma^k(C, c_{-i}^k)\} = \tilde{\mathbf{y}}(x). \quad (4)$$

(3) implies that the quantity and payment pair for each buyer  $i$  depends only on his message but not on the other buyers' messages. For any  $c_{-i}^k \in C^{I-1}$ , recall that  $\gamma^k(C, c_{-i}^k)$  denotes the set of all quantity and payment pairs that each buyer  $i$  can induce from firm  $k$ .

When buyer  $i$  chooses to buy  $x$  from firm  $k$ , there may be many messages that can induce the same quantity  $x$  along with different amounts of payment. If buyer  $i$  ever chooses to buy  $x$  from firm  $k$ , it is always optimal for him to buy  $x$  at the minimum payment. Therefore, the left-hand side of (4) is the minimum payment that the buyer will pay if he trades  $x$  with firm  $k$ . Note that (4) already presumes that firm  $k$  deviates to a mechanism in which the minimum on the left-hand side of (4) exists. In fact, when firm  $k$  deviates to a mechanism satisfying (3) and (4), it is equivalent to offering the price schedule  $\tilde{\mathbf{y}}$ .

Assume that, given firm  $k$ 's deviation to a mechanism satisfying (3) and (4), each buyer  $i$  buys from all firms including the deviating firm. Each buyer  $i$  of type  $\omega_i$  sends the message  $(\eta, \tilde{x}^j(\omega_i))$  to each non-deviating firm  $j$  and sends a message to firm  $k$  in a way that it induces him to buy  $\tilde{x}^k(\omega_i)$  from firm  $k$  at  $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$ . As proved in part (a), each buyer finds it optimal to send  $\eta$  to each non-deviating firm when all the other buyers send the message  $\eta$  to each non-deviating firm. This leads each non-deviating firm to assign the price schedule  $\tilde{\mathbf{y}}$  given its triggering trading mechanism. Because all firms' price schedules, including the deviating firm's, are  $\tilde{\mathbf{y}}$ , it is again optimal for each buyer  $i$  of type  $\omega_i$  to trade  $\tilde{x}^\ell(\omega_i)$  with firm  $\ell$  at  $\tilde{\mathbf{y}}(\tilde{x}^\ell(\omega_i))$  for all  $\ell = 1, \dots, J$ . Parts (a) and (b-1) complete the proof of the first part of Theorem 1.

**(b-2)** Suppose that firm  $k$  deviates to any other trading mechanism  $\gamma^k: C^I \rightarrow X \times \mathbb{R}_+$  that does not belong to (b-1). Suppose that buyer  $i$  is the only one buyer who participates in firm  $k$ 's trading mechanism. Then,  $\gamma^k(C, \emptyset^{I-1})$  is the set of all quantity and payment pairs that buyer  $i$  can choose from firm  $k$  and hence the lowest average price for the buyer becomes

$$p = \inf \left\{ p' \in \mathbb{R}_+ : p' = \frac{m}{x} \text{ for } (x, m) \in \gamma^k(C, \emptyset^{I-1}) \text{ and } x \neq 0 \right\}. \quad (5)$$

We will show that, upon firm  $k$ 's deviation to a trading mechanism  $\gamma^k: C^I \rightarrow X \times \mathbb{R}_+$ , each buyer  $i$  of type  $\omega_i$  buys from only non-deviating firms by sending the message  $(p, \hat{x}(\omega_i))$  to each non-deviating firm, where  $p$  satisfies (5) and  $\hat{x}(\omega_i) \in \arg \max_x u((J-1)x, (J-1)\tau(\mathbf{y})(x), \omega_i)$ . When every buyer

reports  $p \in \mathbb{R}_+$  to each non-deviating firm, the non-deviating firm's price schedule becomes  $\tau(p)$  according to (D1) so that the buyer pays  $\tau(p)(x) = ax$  for any  $x$  that the buyer buys from the non-deviating firm. We first show that it is optimal for each buyer to truthfully report  $p$  defined in (5) to each non-deviating firm when the other buyers do the same.

Assume that all buyers truthfully report  $p$  defined in (5) to each non-deviating firm upon firm  $k$ 's deviation to  $\gamma^k: C^I \rightarrow X \times \mathbb{R}_+$ . Suppose that a buyer reports  $p''$  ( $p'' \neq p$ ) to any non-deviating firm given that all other buyers report  $p$ . If  $I \geq 3$ , then the non-deviating firm's price schedule is still  $\tau(p)$  according to (D1) because still more than half of participating buyers report  $p$ . Therefore, the buyer has no incentive to deviate away from  $p$ . If  $I = 2$ , then the non-deviating firm's price schedule becomes  $\tilde{\mathbf{y}}$  according to (D2) because one buyer reports  $p$  and the other buyer reports  $p''$ . Subsequently, the buyer pays  $\tilde{\mathbf{y}}(x)$  for any  $x$  that the buyer buys from the non-deviating firm. Because of (2),  $\tau(p)$  satisfies  $\tau(p)(x) = ax \leq \tilde{\mathbf{y}}(x)$  for any  $x$ . Hence even when  $I = 2$ , it is optimal for a buyer to truthfully report  $p$  to each non-deviating firm given that the other buyer does the same.

Finally we will show that it is optimal for each buyer to trade  $\hat{x}(\omega_i)$  only with each non-deviating firm. Suppose that buyer  $i$  currently buys  $x$  from a non-deviating firm given that all buyers report  $p$  to the non-deviating firm and that he is the only buyer who buys from the deviating firm. Let  $x'$  be the quantity that buyer  $i$  buys from the deviating firm. Then, the total payment associated with buying  $x$  from the non-deviating firm and  $x'$  from the deviating firm is no less than  $ax + px'$  because of the definition of  $\tau(p)$  in (2) and the definition of  $p$  in (5). However, if the buyer buys  $x + x'$  only from the non-deviating firm, the monetary payment is  $a(x + x')$ , which is no more than  $ax + px'$  because of (2). It implies that the buyer can buy  $x + x'$  with the same or less amount of monetary payment when he buys only from the non-deviating firm. Therefore, it is optimal for each buyer not to buy from the deviating firm when all the other buyers do not buy from the deviating firm. Because each non-deviating firm's price schedule is the linear price schedule  $\tau(p)$ , each buyer  $i$  of type  $\omega_i$  optimally trades the equal quantity with each non-deviating firm by sending  $(p, \hat{x}(\omega_i))$  to it. This completes the proof of the second part of Theorem 1.

When firm  $k$  deviates to a trading mechanism that belongs to (b-1), it receives the same expected profit  $V^k(\tilde{\mathbf{y}})$  that it would receive with the triggering trading mechanism. When firm  $k$  deviates to any other mechanism, i.e., one that belongs to (b-2), it receives its reservation profit  $v_j^j$  because no buyers buy from firm  $k$  in truthful continuation equilibrium. Because the expected profit  $V^k(\tilde{\mathbf{y}})$  associated with the triggering trading mechanism is no less than

$v_o^j$ , firm  $k$  cannot gain by deviating to any alternative mechanism. ■

When all firms maintain their triggering trading mechanisms, their price schedules are  $\tilde{\mathbf{y}}$  in truth-telling continuation equilibrium. When a firm deviates to an arbitrary mechanism that is essentially equivalent to offering  $\tilde{\mathbf{y}}$  to each buyer independent of the other buyers' messages, non-deviating firms do not punish the deviating firm and their price schedules continue to be  $\tilde{\mathbf{y}}$  in truth-telling continuation equilibrium. If a firm deviates to any other mechanism, then each buyer reports, to each non-deviating firm, the lowest average price  $p$  that the deviating firm's mechanism could induce if he participated in the deviating firm's trading mechanism alone, in truth-telling continuation equilibrium. Subsequently, each non-deviating firm offers a linear price schedule that has the unit price equal to the minimum between the average unit price of  $\tilde{\mathbf{y}}$  and  $p$ . This makes it optimal for buyers not to buy from the deviating firm. Therefore, no firm  $j$  can find a profitable deviation to any trading mechanism as long as the firm's expected profit  $V^j(\tilde{\mathbf{y}})$  associated with a price schedule  $\tilde{\mathbf{y}}$  is no less than  $v_o^j$ .

## 5 Discussion

The triggering trading mechanism features convenience in a large class of applications for non-exclusive trading problems under adverse selection. Each buyer's message is two numbers (the deviating principal's lowest average price and the quantity that the buyer wants to buy) and hence communication is simple and independent of the number of buyers while the recommendation mechanism requires the a buyer report his type and the entire mapping of a direct mechanism. The triggering trading mechanism also works for any multiple number of buyers, including the case of two buyers, and the set of equilibrium payoffs is defined in terms of each firm's reservation profit which is independent of trading mechanisms. Let us discuss several important aspects of our result and model.

### **Continuation Equilibrium that Punishes the Deviator**

When firms involve in negotiation with buyers given their negotiation schemes, i.e., mechanisms, buyers know whether some firm deviates. There may be multiple continuation equilibria that buyers can reach upon a firm's deviation. The characterization of equilibrium outcomes in Yamashita (2010) is based on the presumption that buyers then play the (worst) continuation equilibrium that punishes the deviating firm. Because it is a general methodology paper,

it does not show how to derive the continuation equilibrium that can punish the deviating firm upon its deviation to arbitrary mechanism.

Attar, Mariotti and Salanié (2011) discussed how Yamashita's approach can be applied to non-exclusive trading under adverse selection problem. In this setting, firms commit to trading mechanisms in which they sell their products at the competitive unit price when buyers report a firm's deviation. They said that any incentive compatible allocation in which each firm's profit is at least zero can be supported in equilibrium along the lines of Yamashita (2010). However, in order to establish such a result, it is critical to show how to construct continuation equilibrium in which the deviating firm does not gain upon its deviation to any arbitrary trading mechanism; i.e., to show actually what happens to the deviating firm if it deviates to any arbitrary trading mechanism. Because an arbitrary trading mechanism can be quite complex in terms of the degree of communication and pricing, it seems difficult to describe continuation equilibrium off the path following a firm's deviation to any arbitrary trading mechanism. By using triggering trading mechanisms that we propose, our paper completely describes the continuation equilibrium that punishes the deviating firm upon its deviation to any arbitrary trading mechanism.

### **Two or More Buyers**

When non-deviating firms offer the recommendation mechanisms in Yamashita (2010), buyers recommend, to each non-deviating firm, an incentive compatible direct mechanism that each non-deviating firm should implement upon a firm's deviation. Each non-deviating firm implements the recommended direct mechanism only when the majority of buyers recommend the same direct mechanism. Assume that there are three or more buyers. Given that all the other buyers recommend the correct direct mechanism that punishes the deviator, any buyer's unilateral deviation would not change the non-deviating firm's trading mechanism away from the direct mechanism that the other buyers recommend. However, when there are exactly two buyers, the results are ambiguous as Yamashita argued. This is because it is not clear which buyer recommends the correct direct mechanism when two buyers recommend different direct mechanisms. In this case, the non-deviating firm should be able to punish both buyers. However, Yamashita does not further pursue under which circumstances the non-deviating firm can punish both buyers.

Our paper in fact shows that, with much simpler triggering trading mechanism, it is possible to punish both buyers in the two-buyer case when their reports are different in a large class of non-exclusive trading problems under adverse selection. Therefore, implicit collusion among firms is possible as long

as there are at least two buyers.

The difficulty in the two-buyer case is that when their reports on the deviating firm's lowest average price are different, a non-deviating firm does not know which buyer is telling a lie. Suppose that  $p$  is the true lowest average price that the deviating firm's trading mechanism induces when only one buyer participates in its trading mechanism. If one buyer reports something other than  $p$  but the other buyer truthfully reports  $p$ , the triggering trading mechanism continues to offer  $\tilde{y}$  to both of them. This prevents a buyer's lie even in the two-buyer case because a buyer's truthful report, given the other buyer's truthful report, induces the linear price schedule with the unit price that matches the minimum between  $p$  and the lowest average price of  $\tilde{y}$  and this linear price schedule is always better for both buyers than the original price schedule  $\tilde{y}$  that a buyer can induce by his unilateral deviation from  $p$ . Therefore, whether there are two buyers or more, each buyer truthfully reports  $p$  to each non-deviating firm given that the other buyers do the same. Furthermore, it is also optimal for each buyer to buy only from non-deviating firms because the linear price described above always provides better terms of trade for the buyer than the deviating firm's trading mechanism does when he is only one who participates in the deviating firm's trading mechanisms.

### **Two-Way Communication**

The standard approach in competing mechanism design assumes one-way communication in that only agents (e.g., buyers) report messages to principals (firms). One natural question rises on whether principal may benefit from two way communication when he deviates in that he can also send private recommendation to agents similar to the approach adopted by Myerson (1982) for the single principal analysis. Recently, Attar, Campioni and Piaser (2012) proposed competing mechanism games with two-way communication in which each principal offer a mechanism together with private recommendation to each agent. Recommendation could include what type each agent should report to other principals, what quantity each agent should buy from the other principals, and etc. They show that even though competition in incentive compatible direct mechanisms does not generate all equilibrium outcomes from competition in arbitrarily general mechanisms, there is a rationale in restricting attention to incentive compatible direct mechanisms. The reason is that truthful and obedient equilibrium in incentive compatible direct mechanisms is strongly robust in the sense that any continuation equilibrium upon a principal's deviation to an arbitrary mechanism with two-way communication can be replicated by truthful and obedient continuation equilibrium upon a corresponding incentive compatible direct mechanism with two way commu-

nication.<sup>7</sup>

Can a firm then ensure an increase in its profit, given the other firms' triggering trading mechanisms, by deviating to an incentive compatible direct mechanism with two-way communication in non-exclusive contracting problems formulated in our paper? The answer is negative. Suppose that a firm offers a price schedule that lowers its price slightly at every quantity, recommending that all buyers should buy the good from him. However, it is still a continuation equilibrium that all buyers do not follow the deviating firm's recommendation and buy the good only from non-deviating firms by reporting the deviating firm's lowest average price. The key intuition is that generally multiple continuation equilibria exists when a firm deviates to a mechanism and that the multiplicity does not go away even when the deviator's mechanism incorporate two-way communication. Therefore, one can still assign a continuation equilibrium that punishes the deviating firm even if it deviates to mechanisms with two-way communication.

### Heterogenous Buyers

The model in this paper considers a decentralized market where ex-ante identical buyers are located in different places and they look for better deals. The result in Theorem 1 can be easily extended to ex-ante heterogeneous buyers. Let  $u_i(\cdot, \cdot, \omega_i)$  is the payoff function for buyer  $i$  of type  $\omega_i$ . Assume that firms agree to offer an array of price schedules  $[\tilde{\mathbf{y}}_1, \dots, \tilde{\mathbf{y}}_I]$  for buyers, where  $\mathbf{y}_i$  is for buyer  $i$ . Given the price schedule  $\tilde{\mathbf{y}}_i$ , we can find a profile of quantities that buyer  $i$  of type  $\omega_i$  will buy from each firm  $j$ . Given an array of price schedules  $[\tilde{\mathbf{y}}_1, \dots, \tilde{\mathbf{y}}_I]$ , one can construct the triggering trading mechanism for each firm  $j$  that asks each buyer to report a quantity that he wants to buy and an array of the lowest average prices  $[p_1, \dots, p_I]$ , where  $p_i$  is the lowest average price that buyer  $i$  would face if he was the only one who participated in a deviating firm's trading mechanism.

Consider the case in which the number of participating buyers is two or more, and more than half of their reports on some other firm's lowest average prices, one for each buyer  $i$ , are all  $[p_1, \dots, p_I]$  and  $p_i \neq \eta$  for some  $i$ . The triggering trading mechanism then assigns the price schedule  $\tau_i(p_i)$  for buyer  $i$  such that  $\tau_i(p_i)(x) = a_i x$ , where  $a_i = \min \left[ p_i, \inf_{x \in (0, \bar{x}]} \left( \frac{\tilde{\mathbf{y}}_i(x)}{x} \right) \right]$ . In all other cases, the triggering trading mechanism continues to assign  $\tilde{\mathbf{y}}_i$  for each buyer

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<sup>7</sup>Han (2007) and Peters (2001) study the notion of the strongly robust equilibrium. An equilibrium is said to be strongly robust if it survives in all continuation equilibria upon any firm's deviation. Attar, Mariotti, and Salanié (2011) pointed out that strongly robustness is too demanding and especially it may be inconsistent with equilibrium in the market for lemons (i.e., the common-value case).

*i.* Given this triggering trading mechanism, we can show that Theorem 1 is extended for ex-ante heterogeneous buyers.

## 6 Conclusion

The actual contracting or trading process may be quite complex to capture every feature of it in a model. However, implications of the result in this paper are quite significant. Consider a decentralized market where buyers are located in many different places and they search for a better deal in an uncoordinated way. Our paper suggests that once firms sell their products at terms of trade which favor them above the competitive level, it is hard to break them even without their explicit collusive agreement. If a firm lowers its price or offers a better deal for buyers, buyers can reveal such a deviation to the other firms to extract better deals from them while they are negotiating with these non-deviating firms. Therefore, the deviating firm cannot attract buyers as much as it planned. It implies that the good can be overpriced in a competitive decentralized market even with fully rational traders and without any explicit collusive agreement among firms. If an over-priced good (e.g. financial contract in the risk transfer markets) involves non-diversifiable risks in terms of its delivery (e.g. counter-party risks in the risk transfer markets), then a bad turn of an event may trigger a serious downturn in the economy.

It is quite difficult to analyze the market for lemons because risks associated with the delivery of a good are often non-diversifiable and have common-value features in the sense that they affect payoffs of both buyers and sellers. Our paper suggests that in terms of a practical point of view, the market analysis and policy debate for the markets for lemons should in fact carefully examine the actual negotiation process in order to see whether a firm in fact incorporates what it learns from buyers' reports on the competing firms' terms of trade into negotiating its own terms of trade. If they do, firms can sustain collusive outcomes without explicit agreement and, given the common-value nature of risks in the market, the potential adverse effect of overpricing a good on the overall economy can be quite huge.

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