On Take It or Leave It Offers in Common Agency

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Abstract

If the agent's preference relation satisfies a strict monotonicity condition in common agency under the asymmetric information, the set of all equilibrium allocations in the menu game where menus of contracts are allowed coincides with the set of all equilibrium allocations in the single contract game where only single contracts are allowed.

1 Introduction

In common agency problems, competing principals try to control a privately-informed agent's choice. Each principal may offer a single incentive contract that specifies the principal's action as a function of the part of the agent's choice that is contractible by the principal. For example, each principal may specify how much the agent needs to pay for each bundle of goods she buys from the principal or how much the principal will pay the agent as a function of the effort the agent exerts on the principal's tasks and etc.

Alternatively, and more generally, each principal may offer a menu of contracts and let the agent pick one of the contracts and then choose her effort. Theorems 1 and 3 in Peters (2003) showed that if the "no-externalities" condition is satisfied, then, under the *complete information*, the set of *pure-strategy equilibrium* payoffs in the menu game in which menus of contracts are allowed coincides with the set of pure-strategy equilibrium payoffs in the single contract game in which each principal is allowed to offer only a single contract (i.e., take-it-or-leave it offer without negotiation). The "no externalities" condition (Peters 2003, 2007) is satisfied if (i) each principal's utility only depends on his own action and the agent's effort and type, and (ii) conditional on the part of the agent's effort that principal j can contract on, the agent has a *weak preference ordering* over principal j's actions that is independent

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of her payoff type, the part of her effort that principal j cannot contract on, and the other principals' actions.

Peters and Troncoso Valverde (2009) pointed out that the role of mixed strategies has been widely recognized in the literature on common agency and hence it is important to deal with randomization. For example, a seller (principal) may use a mixed-strategy for his trading mechanism offer in order to conceal his mechanism from competing sellers. In this case, competing sellers cannot know the seller's terms of trade if they ask agents only about their payoff types.

We extend the result in Peters (2003, 2007) to all equilibria including *mixed-strategy equilibria* in the *asymmetric information* case in which the agent's payoff type is her own private information. For this purpose, we introduce the condition for the agent's preferences; strict monotonicity over each principal's actions. This condition is satisfied if, conditional on the part of the agent's effort that principal j can contract on, the agent has a *strict preference ordering* over principal j's actions that is independent of her payoff type, the part of her effort that principal j cannot contract on, and the other principals' actions. Therefore, it is a stronger form of the second part in the "no-externalities" condition but it is not nested into the "no-externalities" condition because the first part is not required.

Formally, we show that the strict monotonicity for the agent's preferences alone ensures that under the asymmetric information, the set of all equilibrium allocations in the menu game coincides with the set of all equilibrium allocations in the single contract game. As the result in Peters holds for both public common agency and private common agency, the result is also established for both public common agency and private common agency.[1]

Both the second part in the "no-externalities" condition and our strict monotonicity are satisfied in most cases where each principal's action is *monetary transfer* between him and the agent and they do not require the quasilinearity or additive separability. Later we highlight the relationship between the "no-externalities" condition and the strict monotonicity of the agent's preferences over each principal's actions and explain why the first part of the "no-externalities" condition is not needed.

2 Model

Let us explain the general structure for common agency models. There are a set of principals, $\mathcal{J} \equiv \{1, \cdots, J\}$, and a single agent. The agent has private information about her preferences. This information is parameterized by an element, called a (payoff) type, in a set Ω. Principals do not know the true type but they share a common prior belief that the agent's type follows a probability distribution F on Ω. Therefore, there is the asymmetric information on the agent's type. The agent can take an effort e from a set E. Each principal j can take an action y_j from a set Y_j. We assume that the sets E and Y_j are both compact metric spaces. If the agent of type ω takes an effort e and the array of actions that principals take is $(y_1, \dots, y_J)$, the agent's payoff is $u(y_1, \dots, y_J, e, \omega) \in \mathbb{R}$ and principal j's payoff is $v_j(y_1, \dots, y_J, e, \omega) \in \mathbb{R}$.

[1] In public common agency, the whole effort of the agent is contractible between the agent and a principal. In private common agency, only a part of the agent's effort is contractible between the agent and a principal.

An incentive contract $a_j \colon E \to Y_j$ that principal j offers to the agent specifies his action as a function of the part of the agent's effort that is contractible between them. Following Peters (2003), let $\mathcal{E}_j$ be a collection of measurable equivalence classes, whose union is E such that principal j is constrained to respond to each effort in the same equivalence class the same way. It implies that, for any incentive function a_j, $a_j(e) = a_j(e')$ if e and e' belongs to the same equivalence class, say $\hat{e}$: i.e., $e, e' \in \hat{e}$.

The set of feasible incentive contracts for principal j is therefore defined as $\mathcal{A}_j \equiv \{a_j \in A_j \colon a_j \text{ is } \mathcal{E}_j\text{-measurable}\}$. $\mathcal{A}_j$ differs in the part of the agent's effort that is contractible between principal j and the agent. Let $\mathcal{A} \equiv \times_{k=1}^{J} \mathcal{A}_k$. In public common agency (e.g. lobbying), principal j (lobbyist) can make his action contingent on the whole effort (policy) taken by the agent (policy maker). In this case, each effort e is an equivalence class and any equivalence class is a singleton. In private common agency (e.g., private good trading), an effort e chosen by the agent (buyer) is decomposed into J components, $e = [e_1, \ldots, e_J]$, and principal j (seller) can make his action (monetary transfer) contingent on only the jth component (quantity or quality), e_j, of the agent action. In this case, an equivalence class is the set of the agent's efforts that have the same jth component and it makes an incentive contract a_j effectively specify principal j's action only conditional on e_j.

Principals compete with each other by offering the agent menus of incentive contracts. A menu for principal j is a mapping $\gamma_j \colon \mathcal{A}_j \to \mathcal{A}_j$ such that either $a_j = \gamma_j(a_j)$ or $\bar{a}_j = \gamma_j(a_j)$ for some $\bar{a}_j \in \mathcal{A}_j$. The interpretation is that the agent can name the incentive contract that she wants and if it belongs to the menu γ_j gets it. A principal may offer a more complex mechanism in which the incentive contract is determined by the agent's message. However, there is no loss of generality to focus on menus in common agency due to Peters (2001) because the set of all equilibrium payoffs in the complex mechanism game is the same as the set of all equilibrium payoffs in the menu game. Let Γ_j^* be the set of all menus available for principal j. and $\Gamma^* \equiv \times_{k=1}^{J} \Gamma_k^*$.

The *menu game* in common agency starts when each principal j simultaneously offers a menu from Γ_j^*. After seeing a profile of menus $\gamma = [\gamma_1, \cdots, \gamma_I] \in \Gamma^*$, the agent names an incentive contract for each principal and takes her effort from E. The agent's effort then determines principals' actions given the incentive functions that the agent gets. Finally, payoffs are realized. The agent's continuation strategy is a mapping $c \colon \Gamma^* \times \Omega \to \Delta(\mathcal{A} \times E)$. The continuation strategy c^* constitutes a continuation equilibrium if the randomization $c^*(\gamma, \omega)$ maximizes the agent's payoff

$$\int_{\mathcal{A} \times E} u\left(\gamma_1(a_1)(e), \ldots, \gamma_J(a_J)(e), e, \omega\right) dc(\gamma, \omega),$$

where $\gamma_j(a_j)$ is the incentive contract that agent i gets when she names the contract a_j and $\gamma_j(a_j)(e)$ is principal j's action conditional on the contractible part of the effort e that the agent chooses. Then, the continuation equilibrium c^* specifies the normal form game for principals in which each principal j's expected payoff function is, for all $\gamma \in \Gamma^*$

$$V_j(\gamma, c^*) = \int_{\Omega} \int_{\mathcal{A} \times E} v_j\left(\gamma_1(a_1)(e), \ldots, \gamma_J(a_J)(e), e, \omega\right) dc^*(\gamma, \omega) dF.$$

Let $\delta_j \in \Delta(\Gamma_j^*)$ be principal j's strategy. An equilibrium relative to Γ^* is an array of randomization $[\delta_1^*, \ldots, \delta_J^*]$ and a continuation equilibrium c^* such that $[\delta_1^*, \ldots, \delta_J^*]$ is a Nash equilibrium for the normal form game defined by c^*.

3 Main Result

In the single contract game, each principal j is only allowed to offer an incentive contract in $\mathcal{A}_j$. Given an array of incentive contracts offered by principals, the agent simply makes her effort choice. An incentive contract a_j can be interested as a degenerate menu because it is equivalent to the menu γ_j with $a_j = \gamma_j(a_j')$ for all a_j'. Because the set of degenerate menus is a strict subset of Γ_j^*, the single contract game restricts the principal's strategy space. It raises two concerns for the single contract game. First, the equilibrium in the single contract game may disappear once a principal is allowed to offer non-degenerate menus. Second, the single contract game may not generate all equilibrium allocations that could have been generated by the menu game.

Theorem 1 in Peters (2003) shows that under the complete information, any pure-strategy equilibrium in the single contract game continues to be an equilibrium in the menu game. Theorem 3 in Peters (2003) states as follows. Suppose that the "no-externalities" condition holds. Then, under the complete information, payoffs associated with any pure-strategy equilibrium in the menu game are supported by a pure-strategy equilibrium in the single contract game. Therefore, the theorems imply that the "no-externalities" condition ensures that under the complete information, the set of pure-strategy equilibria in the single-contract game coincides with the set of pure-strategy equilibria. The "no-externalities" condition in Peters (2003, 2007) is stated as follows:

D1. For each $j \in \mathcal{J}$, there exists a function $\bar{v}_j : Y_j \times E \times \Omega \to \mathbb{R}$ such that for all $(y_1, \ldots, y_J) \in \times_{k=1}^{J} Y_k$, all $e \in E$, and all $\omega \in \Omega$

$$v_j(y_1, \ldots, y_J, e, \omega) = \bar{v}_j(y_j, e, \omega)$$

D2. For each $j \in \mathcal{J}$, each $\hat{e} \in \mathcal{E}_j$, each closed subset $B_j^* \subset Y_j$, the set

$$\left\{y_j \in B_j^* : u(y_j, y_{-j}, e, \omega) \geq u(y_j', y_{-j}, e, \omega) \text{ for all } y_j' \in B_j^*\right\} \tag{1}$$

is the same for all $(y_{-j}, e, \omega) \in Y_{-j} \times \hat{e} \times \Omega$.

Property D1 states that principal j's payoff is independent of the actions chosen by the other principals. Property D2 states that conditional on the part of the agent's effort that principal j can contract on, the agent has a *weak preference ordering* over principal j's actions that is independent of her payoff type, the part of her effort that principal j cannot contract on, and the other principals' actions. Peters (2003) showed that the agent's weak preference ordering alone is not sufficient in order to ensure that no additional (pure-strategy) equilibrium can be emerged if principals are allowed to offer menus. Let us take his example under the complete information where the agent has no effort and each of two principals has choices between two actions; $\{a_1, b_1\}$ for principal 1 and $\{a_2, b_2\}$ for principal 2.[2] The tuple of three numbers in the cell below indicates payoffs for principal 1, principal 2 and the agent respectively.

	a_2	b_2
a_1	$0, 0, 1$	$2, 1, 1$
b_1	$1, 2, 1$	$3, 3, 0$

In this example, the agent has a weak preference ordering over each principal's actions that is independent of the other principal's action: For the agent, a_1 is at least as good as b_1 regardless of principal 2's action and a_2 is at least as good as b_2 regardless of principal 1's action. However, property D1 is not satisfied.

There is a unique (pure-strategy) equilibrium when each principal i is restricted to offer a_i or b_i but not the menu of $\{a_i, b_i\}$. In the unique equilibrium, principal 1 offers b_1 and principal 2 offers b_2. If we expand the game so that each principal can offer the menu of two actions, then we have a continuum of new equilibria. Suppose that each principal i offers the menu of $\{a_i, b_i\}$. Then the agent may randomize with probability q between the two off diagonal outcomes when she makes her choice from the menus. This supports payoffs for principals anywhere between 1 and 2. If principal i deviates to a_i, then the agent chooses a_j from the other principal's menu and principal i's payoff is zero. If principal i deviates to b_i, then the agent chooses a_j to induce the payoff of one for principal i.

Let us perturb the payoff slightly with a small $\varepsilon > 0$ as follows:

	a_2	b_2
a_1	$0, 0, 1 + \varepsilon$	$2, 1, 1$
b_1	$1, 2, 1$	$3, 3, 0$

In this case, the agent has a strict preference ordering over each principal's actions that is independent of the other principal's action: The agent strictly prefers a_1 to b_1 regardless of principal 2's action and she also strictly prefers a_2 to b_2 regardless of principal 1's action. Therefore, if principal i offers the menu of $\{a_i, b_i\}$, the agent will always choose a_i regardless of the other principal's action. Therefore, b_i in the menu $\{a_i, b_i\}$ will be never chosen in any continuation equilibrium and, even without property D1, there is no additional equilibrium in the menu game that cannot be generated by a take-it-or-leave-it-offer game.

This intuition can be extended to the case with the agent's effort under the asymmetric information. For this purpose, we define the strict monotonicity of the agent's preferences in each principal's action. In words, the agent's preferences are strictly monotone if, conditional on the part of the agent's effort that principal j can contract on, the agent has a *strict preference ordering* over principal j's actions that is independent of her payoff type, the part of her effort that principal j cannot contract on, and the other principals' actions:

[2] As Peck (1997) showed, we can similarly construct an example which shows that without restrictions on preferences, a menu game may generate an additional mixed-strategy equilibrium allocation that cannot be generated by a take-it-or-leave-offer game. However, we take the example in Peters (2003) that shows a menu game generates an additional (pure-strategy) equilibrium allocation. The reason is that it can clearly illustrate the relationship between the "no-externalities" condition and our strict monotonicity of the agent's preferences over each principal's action.

Definition 1 *The agent's preferences are* strictly monotone *in each principal's action if, for each* $j \in \mathcal{J}$*, each equivalence class* $\hat{e} \in \mathcal{E}_j$*, and each compact subset* $B_j^* \subset Y_j$*,*

$$\left\{y_j \in B_j^* : u(y_j, y_{-j}, e, \omega) > u(y_j', y_{-j}, e, \omega) \text{ for all } y_j' \in B_j^* \text{ with } y_j' \neq y_j\right\} \tag{2}$$

is the same for all $(y_{-j}, e, \omega) \in Y_{-j} \times \hat{e} \times \Omega$*.*

The only difference between property D2 in the "no-externalities" condition and our strict monotonicity is that the weak inequality in (1) is replaced with the strict inequality as shown in (2). All the examples for the agent's preferences in Peters (2003) that satisfy property D2 also satisfy our strict monotonicity: Both property D2 and the strict monotonicity (a stronger version of D2) are satisfied in most cases where each principal j's action is *monetary transfer* between him and the agent. For example, assume that the agent is the buyer who buys goods from two sellers (principals) and her utility function is

$$u(y_1, y_2, e_1, e_2, \omega) = \sqrt{\frac{\exp(e_1 e_2)^{\omega}}{(y_1 + y_2)^2}}, \tag{3}$$

where e_j is the quantity of the good that the buyer buys from seller j and y_j is the transfer to seller j. Seller j receives transfer only conditional on the quantity of the good e_j that he sells to the buyer, so, for each e_j, the equivalence class is $\{(e_j, e_i) : e_i \in \mathbb{R}_+\}$. Because, conditional on each e_j, the buyer strictly prefers a less transfer to seller j regardless of e_i, y_i, and ω, the strict monotonicity is satisfied for the agent's preferences.

Both property D2 in the "no-externalities" condition and our strict monotonicity do not require the quasilinearity or the separability and genericity in Attar et al. (2008). Attar et al. (2008) showed that if the separability and genericity are satisfied for the agent's preferences, then the set of all equilibrium allocations in complex mechanisms is the set of all equilibrium allocations in standard direct mechanisms in the environment with *finite action* space. Their separability requires that the agent has the strict preference ordering over each principal's action y_j that is independent of her *whole effort* e and the other principals' actions y_{-j}.

Formally the separability in Attar et al. (2008) requires that if the agent of type ω strictly prefers y_j to y_j' given (y_{-j}, e), then she also strictly prefers y_j to y_j' given any other (y_{-j}', e'). The genericity requires that given any $(y_j, \omega) \in Y_j \times \Omega$, $u(y_j, y_{-j}, e, \omega) \neq u(y_j, y_{-j}', e', \omega)$ for any $(y_{-j}, e), (y_{-j}', e') \in Y_{-j} \times E$. The separability in Attar et al. (2008) is not weaker nor stronger than property D2 of the "no-externalities" condition and the strict monotonicity in our paper[3], but Pavan and Calzolari (2010) showed that the separability is restrictive in most common agency problems involving monetary transfer. One advantage of property D2

[3]The separability requires that the agent's preferences over each principal j's action to be independent of the agent's whole effort e but property D2 in the "no-externalities" condition and our strict monotonicity require them to be independent of the particular effort the agent chooses in a given equivalence class for principal j (in other words, the part of effort that principal j cannot contract on). On the other hand, property D2 in the "no-externalities" condition and our strict monotonicity requires the agent's preferences over each principal's action be independent of the agent's type while such a dependence is allowed in the separability.

in the "no-externalities" condition and our strict monotonicity (a stronger version of D2) is that they are satisfied in most cases where each principal's action is monetary transfer.

With the strict monotonicity, we extend the results in Peters (2003) for all equilibrium allocations under the asymmetric information as follows.

Theorem 1 *Suppose that the agent's preferences are strictly monotone in each principal's action. Then, under the asymmetric information, the set of all equilibrium allocations relative to the single contract game is the same as the set of all equilibrium allocations in the menu game.*

Note that Theorem 1 is established by assuming that for each principal j, the sets of feasible contracts in the single contract game and the menu games are all $\mathcal{A}_j$ and it is $\mathcal{E}_j$-measurable. Because $\mathcal{A}_j$ is $\mathcal{E}_j$-measurable, in public common agency it includes all incentive contracts that specify principal j's action conditional on the agent's effort e as a whole. In private common agency, principal j can specify his action conditional only on the jth component e_j of the agent's effort. Therefore, the $\mathcal{E}_j$-measurable $\mathcal{A}_j$ includes only those incentive contracts that effectively specify principal j's action conditional on e_j only. Therefore, as the result in Peters (2003) holds for both public common agency and private common agency, Theorem 1 is also established for both public common agency and private common agency. Appendix includes omitted technical details including the proof of Theorem 1.

APPENDIX

We first develop a few notations for the proof of Theorem 1. For any $j \in \mathcal{J}$, any $\gamma_j \in \Gamma_j^*$, and any $\hat{e} \in \mathcal{E}_j$ define $B_j(\hat{e}, \gamma_j)$ as

$$B_j(\hat{e}, \gamma_j) \equiv \{y_j \in Y_j \colon y_j = \gamma_j(a_j)(e) \text{ for all } a_j \in \mathcal{A}_j \text{ and any } e \in \hat{e}\}. \tag{4}$$

$\gamma_j(a_j)$ is the incentive function that principal j assigns when the agent names the incentive contract a_j so that $B_j(\hat{e}, \gamma_j)$ is the set of principal j's actions that the agent can induce when she takes any effort e in an equivalence class $\hat{e}$. For all $j \in \mathcal{J}$, all $\gamma_j \in \Gamma_j^*$, and all $e \in E$, let

$$\psi_j(\gamma_j)(e) \equiv \underset{y_j \in B_j(\hat{e}, \gamma_j)}{\arg\max}\; u(y_j, y_{-j}, e, \omega) \tag{5}$$

be principal j's action that maximizes the agent's payoff among all actions in $B_j(\hat{e}, \gamma_j)$ when she takes e, where $\hat{e}$ in (5) is the equivalence class that satisfies $e \in \hat{e}$. When the agent's preference relation is strictly monotone in each principal's action, $\psi_j(\gamma_j)(e)$ is a singleton for all $e \in E$ so that $\psi_j(\gamma_j)$ itself becomes an incentive contract that specifies principal j's action as a function of the part of the agent's effort that he can contract on.

For technical simplicity, we assume that $\psi_j(\gamma_j)(e)$ is non-empty for all $j \in \mathcal{J}$, all $e \in E$, and all $\gamma_j \in \Gamma_j^*$

Lemma 1 *Suppose that the agent's preference relation is strictly monotone in each principal's action. For any continuation equilibrium c^* relative to Γ^*, let $c_a^*(e, \gamma, \omega)$ denote the probability distribution over any array of incentive contracts conditional on $(e, \gamma, \omega) \in E \times \Gamma^* \times \Omega$. Then any $(a_1, \ldots, a_J)$ in the support of $c_a^*(e, \gamma, \omega)$ satisfies*

$$\gamma_j(a_j)(e) = \psi_j(\gamma_j)(e) \tag{6}$$

for all $(e, \gamma, \omega) \in E \times \Gamma^* \times \Omega$ and all $j \in \mathcal{J}$.

Proof. Let $\gamma = [\gamma_1, \ldots, \gamma_J]$ be the profile of menus that principals offer. Given a continuation equilibrium c^* relative to Γ^*, let $c^*_{a_j}(e, \gamma, \omega)$ be the marginal probability distribution on $\mathcal{A}_j$ conditional on (e, γ, ω). Suppose that the agent's preference relation is strictly monotone in each principal's action. Given γ_j, the set of principal j's actions, $B_j(\hat{e}, \gamma_j)$ defined in (4), that the agent can induce depends on effort e that she takes because e subsequently determines $\hat{e}$. Once $B_j(\hat{e}, \gamma_j)$ is determined, the agent will always choose a message that leads to $\psi_j(\gamma_j)(e) \in B_j(\hat{e}, \gamma_j)$ because of the strict monotonicity of the agent's preference relation. It implies that given e and γ_j, any a_j in the support of $c^*_{a_j}(e, \gamma_j, \gamma_{-j}, \omega)$ must satisfy (6) regardless of γ_{-j} and ω. Therefore, any $(a_1, \ldots, a_J)$ in the support of $c^*_{a_j}(e, \gamma, \omega)$ satisfies (6) for all $(e, \gamma, \omega) \in E \times \Gamma^* \times \Omega$ and all $j \in \mathcal{J}$. ■

Proof of Theorem 1. First start with an equilibrium $[\delta^*_1, \ldots, \delta^*_J, c^*]$ relative to Γ^*. The equilibrium profile of strategies $[\delta^*_1, \ldots, \delta^*_J, c^*]$ induces the probability distribution over $E \times Y$ conditional on the agent's type. Let $\pi^* \colon \Omega \to \Delta(E \times Y)$ specify these conditional distributions associated the equilibrium $[\delta^*_1, \ldots, \delta^*_J, c^*]$. In the mechanism design literature, π^* is called a social choice function and it characterizes the allocation in the economy.

For each $j \in \mathcal{J}$, let $\tilde{\sigma}_j$ be the probability measure over $\mathcal{A}_j$ that is induced by δ^*_j through the map ψ_j. It is the mixed strategy that principal j will use in the single contract game (i.e., the game relative to $\mathcal{A}$). Let ψ_j^{-1} be the inverse correspondence of ψ_j. For any $a_j \in \mathcal{A}_j$, define the set $D_j(a_j) \subset \Gamma^*_j$ as

$$D_j(a_j) \equiv \begin{cases} \psi_j^{-1}(a_j) \cap \text{ supp } \delta^*_j & \text{if } \psi_j^{-1}(a_j) \cap \text{ supp } \delta^*_j \neq \varnothing \\ \bar{\psi}_j^{-1}(a_j) & \text{otherwise,} \end{cases}$$

where $\bar{\psi}_j^{-1}(a_j)$ is an arbitrary menu in $\psi_j^{-1}(a_j)$. For any $a = [a_1, \ldots, a_J] \in \mathcal{A}$, let $D(a) = \times_{k=1}^{J} D_k(a_k) \subset \Gamma^*$.

From the equilibrium strategy profile $[\delta^*_1, \ldots, \delta^*_J, c^*]$ relative to Γ^*, we can derive a joint probability distribution $b(D, \omega)$ on $\mathcal{A} \times E$ for all $D \subset \Gamma^*$ and all $\omega \in \Omega$. Let $b_a(e, D, \omega)$ be the probability distribution on $\mathcal{A}$ conditional on (e, D, ω) that $b(D, \omega)$ induces. Let $b_e(D, \omega)$ be the marginal probability distribution on E that $b(D, \omega)$ induces. Construct the agent's continuation strategy $\tilde{c} : \mathcal{A} \times \Omega \to \Delta(E)$ relative to $\mathcal{A}$ as

$$\tilde{c}(a, \omega) = b_e(D(a), \omega) \tag{7}$$

for all $(a, \omega) \in \mathcal{A} \times \Omega$. We will show that $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ is an equilibrium relative to $\mathcal{A}$. Note that Lemma 1 implies that any $(a_1, \ldots, a_J)$ in the support of $b_a(e, D(a), \omega)$ induces the same profile of principals' actions, $[a_1(e), \ldots, a_J(e)] \in Y$. Therefore, (7) ensures that the social choice function $\pi^* \colon \Omega \to \Delta(E \times Y)$ associated with the equilibrium $[\delta^*_1, \ldots, \delta^*_J, c^*]$ relative to Γ^* is the same as the social choice function $\tilde{\pi} \colon \Omega \to \Delta(E \times Y)$ associated with $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ relative to $\mathcal{A}$.

Given $\gamma \in D(a)$ and her payoff type $\omega \in \Omega$, the agent's optimal choice of her effort then satisfies

$$e \in \arg\max_{x' \in X} u\left(a_1(e'), \ldots, a_1(e'), e', \omega\right). \tag{8}$$

Any e in the support of $b_e(D(a),\omega)$ satisfies (8) because the joint probability distribution $b(D(a),\omega)$ is derived from the continuation equilibrium c^* relative to Γ^*. Therefore, (7) implies that $\tilde{c}$ is a continuation equilibrium relative to $\mathcal{A}$.

We only need to show that $\tilde{\sigma}_j$ is a best response for principal j given $\tilde{\sigma}_{-j}$ relative to $\mathcal{A}$. Consider each principal j's payoff. For any $(a_j, a_{-j}) \in \mathcal{A}$, let

$$\begin{aligned} v_j^*(a_j, a_{-j}, \tilde{c}) &= \int_\Omega \int_E v_j\left(a_j(e), a_{-j}(e), e, \omega, \right) d\tilde{c}(a,\omega) dF \\ &= \int_\Omega \int_E v_j\left(a_j(e), a_{-j}(e), e, \omega, \right) db_e(D(a),\omega) dF \\ &= \mathbb{E}_{\gamma\in D(a)} \left[\int_\Omega \int_{\mathcal{A}\times E} v_j\left(a_j(e), a_{-j}(e), e, \omega\right) dc^*(\gamma,\omega) dF \right]. \end{aligned}$$

Integrating $v_j^*(a_j, a_{-j}, \tilde{c})$ using $\tilde{\sigma}_{-j}$ yields

$$\begin{aligned} & V_j(a_j, \tilde{\sigma}_{-j}, \tilde{c}) \\ = \ & \mathbb{E}_{\gamma_j\in D_j(a_j)} \left[\int_{\Gamma^*_{-j}} \int_\Omega \int_{\mathcal{A}\times E} v_j\left(a_j(e), a_{-j}(e), e, \omega\right) dc^*(\gamma_j, \gamma_{-j}, \omega) dF d\delta_{-j} \right] \\ = \ & \mathbb{E}_{\gamma_j\in D_j(a_j)} [V_j(\gamma_j, \delta^*_{-j}, c^*)]. \end{aligned} \tag{9}$$

First, consider any $a_j \in \text{supp } \tilde{\sigma}_j$. The construction of $\tilde{\sigma}_j$ implies that $D_j(a_j) = \psi_j^{-1}(a_j) \cap$ supp $\delta_j^* \neq \varnothing$. Because any $\gamma_j \in D_j(a_j)$ is in the support of $\tilde{\delta}_j$, (9) implies that, for all $\gamma_j \in D_j(a_j) = \psi_j^{-1}(a_j) \cap$ supp δ_j^*,

$$V_j(a_j, \tilde{\sigma}_{-j}, \tilde{c}) = V_j(\gamma_j, \delta^*_{-j}, c^*) = V_j(\delta^*_j, \delta^*_{-j}, c^*). \tag{10}$$

Second, consider any $\hat{a}_j \notin \text{supp } \tilde{\sigma}_j$. Then $\psi_j^{-1}(\hat{a}_j) \cap$ supp $\delta_j^* = \varnothing$. In this case, $D_j(\hat{a}_j)$ is a singleton of $\bar{\psi}_j^{-1}(\hat{a}_j) \in \Gamma_j^*$ and (9) implies that

$$V_j(\hat{a}_j, \tilde{\sigma}_{-j}, \tilde{c}) = V_j(\bar{\psi}_j^{-1}(\hat{a}_j), \delta^*_{-j}, c^*). \tag{11}$$

Because $[\delta_1^*, \ldots, \delta_J^*, c^*]$ is an equilibrium relative to Γ^*, $V_j(\delta^*_j, \delta^*_{-j}, c^*) \geq V_j(\bar{\psi}_j^{-1}(\hat{a}_j), \delta^*_{-j}, c^*)$. Therefore, (10) and (11) imply that $\tilde{\sigma}_j$ is a best response for principal j when the other principals use $\tilde{\sigma}_{-j}$ given a continuation equilibrium $\tilde{c}$. Therefore, $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ is an equilibrium relative to $\mathcal{A}$.

Now start with an equilibrium $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ relative to $\mathcal{A}$. Let $\tilde{\pi}\colon \Omega \to \Delta(E\times Y)$ be the social choice function that is supported by $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$. Note that any incentive function a_k can be viewed as a degenerate menu γ_k that assigns a_k regardless of the menu that the agent names. For principal j's deviation to mechanisms in Γ_j^*, one can associate c', due to by Lemma 1, with a continuation equilibrium strategy $c' : \Gamma_j^* \times \mathcal{A}_{-j} \times \Omega \to \Delta(\mathcal{A}_j \times X)$ relative to $\Gamma_j^* \times \mathcal{A}_{-j}$ as follows. The probability distribution $c'_{a_j}(e, \gamma_j, a_{-j}, \omega)$ on $\mathcal{A}_j$ satisfies, for all $a_j \in \text{supp } c'_{a_j}(e, \gamma_j, a_{-j}, \omega)$,

$$\gamma_j(a_j)(e) = \psi_j(\gamma_j)(e) \tag{12}$$

and the probability distribution $c'_e(\gamma_j, a_{-j}, \omega)$ on E satisfies

$$c'_e(\gamma_j, a_{-j}, \omega) = \tilde{c}(\psi_j(\gamma_j), a_{-j}, \omega). \tag{13}$$

If principal j deviates to a menu γ_j in Γ^*_j, his payoff becomes

$$V_j(\gamma_j, \tilde{\sigma}_{-j}, c') = V_j(\psi_j(\gamma_j), \tilde{\sigma}_{-j}, \tilde{c}) \tag{14}$$

because of (13). Because $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ is an equilibrium relative to $\mathcal{A}$ and $\psi_j(\gamma_j) \in \mathcal{A}_j$, we have

$$V_j(\tilde{\sigma}_j, \tilde{\sigma}_{-j}, \tilde{c}) \geq V_j(\psi_j(\gamma_j), \tilde{\sigma}_{-j}, \tilde{c}). \tag{15}$$

Combining (14) and (15) yields $V_j(\tilde{\sigma}_j, \tilde{\sigma}_{-j}, \tilde{c}) \geq V_j(\gamma_j, \tilde{\sigma}_{-j}, c')$. Therefore, $[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_J, \tilde{c}]$ is also an equilibrium relative to Γ^* and hence the social choice function $\tilde{\pi} \colon \Omega \to \Delta(E \times Y)$ continues to be supported. ■

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