Abstract

This paper investigates mediated communication between an informed sender and an uninformed receiver with conflicting preferences in the framework of Crawford and Sobel (1982). It provides a simple condition for mediation to be beneficial, that is, to give the receiver a higher ex-ante payoff than the uninformed decision. This condition in turn allows us to identify scenarios in which mediation is beneficial while all cheap-talk equilibria are uninformative. Our condition extends the conditions for beneficial mediation with a binary type space (Mitusch and Strausz, 2005) and mediation via a biased mediator (Ambrus et al., 2013). Finally, we show the connection between the identified condition and related conditions in other conflict resolution schemes: delegation (Alonso and Matouschek, 2008) and arbitration (Kovác and Mylovanov, 2009).

JEL classification: C72, D81, D82, D83

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1 Introduction

Beginning with the classical work by Crawford and Sobel (1982), hereafter CS, the literature on cheap talk (or direct talk) emphasizes that conflict of interest is the main source of ineffective communication between an informed agent (the sender) and an uninformed decision maker (the receiver). Moreover, if this conflict is large, meaningful communication between the interacting parties is not feasible.¹ In this case, the parties must use alternative schemes of conflict resolution that facilitate communication. A common scheme is using

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¹Department of Economics, McMaster University, 1280 Main Street West, Hamilton, ON, Canada L8S 4M4. E-mail: mivanov@mcmaster.ca. Phone: +1-905-525-9140x24532. Fax: +1-905-521-8232.

¹Krishna and Morgan (2001) and Ambrus and Takahashi (2008) show that communication can be facilitated by consulting multiple senders with conflicting preferences. However, if the bias between their preferences and that of the receiver is large, all equilibria are outcome-equivalent to the uninformative one.
a neutral (i.e., non-strategic) mediator who is initially not informed about the issue and cannot enforce his recommendations. The primary goal of the mediator is to privately obtain information from the sender and to give private advice to the receiver. By properly distorting information received from the sender, the mediator may be able to provide the sender the incentive to reveal more information.

However, if the conflict of interest is so intense that direct talk is uninformative, it is not clear whether introducing a mediator can facilitate communication. Moreover, the mediator’s participation often requires costs or effort from the conflicting parties. Hence it is important to identify scenarios in which: 1) all equilibria in the direct-communication game are uninformative, and 2) mediation is beneficial in that it improves upon the uninformed outcome. As to 1), CS provide a necessary and sufficient condition for the existence of uninformative equilibria only. Regarding 2), the existing literature provides conditions under which various conflict resolution and communication schemes are beneficial, e.g., delegation (Alonso and Matouschek, 2008), arbitration (Kovác and Mylovanov, 2009), communication via the strategic mediator (Ambrus et al., 2013), and cheap talk with a possibly uninformed sender (Austen-Smith, 1994). But this has remained an open question for non-strategic mediation. This issue forms the central focus of our work.

Our results are as follows. First, we provide a sufficient condition for beneficial mediation in the CS framework. This condition is intuitively clear and can be easily verified. It requires the existence of a cutoff type such that a sender of this type strictly prefers the receiver’s uninformed decision (the one based on prior information only) to the decision the receiver would take if told that the true type lies on a particular side of this cutoff, that is, above or below it. In particular, this condition holds if there exists a type whose ideal decision coincides with the uninformed decision. Second, we show that this condition is always satisfied when the mediation game admits an equilibrium with a partition of the type space into two subintervals; in this case, the type at the boundary between these intervals has the cutoff property. Third, by combining this condition with the CS condition for uninformative cheap talk, we identify scenarios in which mediation improves strictly upon direct communication.

Fourth, we show the connection between our condition and related conditions in alternative schemes of conflict resolution: communication via the biased mediator (Ambrus et al., 2013), delegation (Alonso and Matouschek, 2008), and arbitration (Kovác and Mylovanov, 2009). In particular, our condition extends sufficient conditions by Ambrus et al. (2013), which are imposed on the sender’s lowest type, to arbitrary types. It is, however, stronger than the condition of minimally aligned preferences in delegation (Alonso and Matouschek, 2008). Intuitively, additional restrictions in mediation stem from the optimality

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2 Because a neutral mediator is not interested in the outcome of the game, he is equivalent to a mediation mechanism that commits to a certain algorithm of processing and transmitting information.

3 In delegation, the receiver commits to actions contingent on the sender’s messages. In arbitration, the mediator’s recommendations to the receiver are binding.

4 Thus, we do not rely on the uniform-quadratic specification used in most of the literature on mediated communication (Blume et al., 2007; Goltsman et al., 2009; and Ivanov, 2010). To the best of our knowledge, the only papers on mediation beyond the uniform-quadratic case are Ambrus et al. (2013) and Alonso and Rantakari (2013). Also, Austen-Smith (1994) considers the CS framework with a possibly uninformed sender. However, all equilibria in that setup are outcome-equivalent to equilibria in mediation with the perfectly informed sender.

5 However, our condition does not guarantee an improvement over informative direct communication.
of the receiver’s response to any relevant information. Finally, our condition is also equivalent to the necessary and sufficient condition for beneficial arbitration introduced by Kováč and Mylovanov (2009) under their regularity condition.

Also, our condition combines the two sufficient conditions for beneficial mediation by Mitusch and Strausz (2005) in the case of the binary set of sender’s types and extends them to the continuous type space. This extension is not straightforward. The first difficulty is that Mitusch and Strausz (2005) impose conditions on the binary prior distribution that cannot be applied to other discrete distributions. Another difficulty stems from the fact that the incentives of any sender’s type in continuous-type models cannot be separated from those of nearby types. In particular, in the search for the sender’s cutoff type with certain preferences over receiver’s actions, we must take into account that these actions are affected by strategies of the sender’s types above and below the cutoff type. In turn, the receiver’s actions influence the preferences of the cutoff type. Clearly, this cyclical problem does not appear in the binary-type case.

The rest of the paper is structured as follows. Section 2 presents the formal model. In Section 3, we provide the sufficient condition for beneficial mediation and characterize scenarios in which mediation is beneficial while cheap talk is uninformative. Section 4 establishes the connection between our condition and related conditions in other conflict resolution schemes. Section 5 concludes the paper.

2 The Model

Our model is based on the CS framework. There are two players in the game, a privately informed sender \((S)\) and an uninformed receiver \((R)\). The sender perfectly knows state \(\theta\), which is distributed according to a continuous distribution function \(F\) with a density \(f > 0\) on the support \(\Theta = [0, 1]\). The receiver takes a decision (or action) \(y \in \mathbb{R}\). The preferences of player \(i \in \{S, R\}\) are given by the payoff function \(U_i(y, \theta)\), which is strictly concave in \(y\) for each \(\theta\), continuous and strictly supermodular in \((y, \theta)\), and achieves the maximum at the ideal decision \(y_i(\theta)\) for all \(\theta \in \Theta\). This implies that \(y_i(\theta), i \in \{S, R\}\) is unique and bounded for all \(\theta \in \Theta\), and continuous and strictly increasing in \(\theta\).

If the receiver holds a posterior belief \(\mu \in \Delta \Theta\), his interim payoff

\[ EU_R(y|\mu) = E_{\mu} [U_R(y, \theta)], \]

has the unique maximizer \(y^*(\mu)\). For \(s < t\), consider the receiver’s interim payoff given the belief that \(\theta \in [s, t]\),

\[ EU_R(y|s, t) = \frac{1}{F(t) - F(s)} \int_s^t U_R(y, \theta) f(\theta) d\theta, \]

\(^6\)Because sender’s types are isolated from each other in the discrete-type setup, there exists the fully separating equilibrium if the conflict of interest is small enough. This is never the case with a continuum of types unless the players’ preferences are perfectly aligned.

\(^7\)A function \(U(y, \theta)\) is strictly supermodular if \(U(y'', \theta'') - U(y'', \theta') > U(y', \theta'') - U(y', \theta')\) for all \(y'' > y', \theta'' > \theta'\).
and the maximizer of $EU_R(y|s, t)$,
$$
y^t_s = \arg \max_{y \in \mathbb{R}} EU_R(y|s, t).
$$

Let
$$
y^*_z(p) = \arg \max_{y \in \mathbb{R}} pEU_R(y|0, z) + (1 - p) EU_R(y|z, 1)
$$
be the maximizer of the receiver’s interim payoff given the belief that $\theta \in [0, z]$ with probability $p$ and $\theta \in [z, 1]$ with probability $1 - p$. The uniqueness of $y^*_z(p)$ implies that it is continuous in $p$ (Sydsæter et al., 2005, p. 103).

**Strategies and mediation rules.** The timing of the game is as follows. First, the sender observes $y$ and sends a signal $s$ to the mediator. The mediator then sends a message $m$ to the receiver, who takes a decision $y$. Thus, the sender’s strategy $\xi : \Theta \to \Delta S$ is a measurable mapping from $\Theta$ into the set of probability distributions over a measurable signal space $S \supset \Theta$. A mediation rule $\sigma : S \to \Delta M$ is a measurable mapping from the signal space into the set of probability distributions over a measurable message space $M \supset \mathbb{R}$. The receiver’s strategy $y : M \to \mathbb{R}$ specifies the decision $y(m) \in \mathbb{R}$ as a function of the mediator’s message $m$.

**Equilibrium.** We can employ the Revelation Principle and restrict attention to direct truth-telling equilibria. In these equilibria, the sender truthfully reports her type to the mediator, and the receiver follows the mediator’s recommendation. That is, the mediation rule in a direct truth-telling equilibrium is a mapping $\sigma : \Theta \to \Delta \mathbb{R}$ from the state space $\Theta$ into the set of probability distributions over the action space $\mathbb{R}$.

For each sender’s report $\theta$, a mediation rule induces a distribution over recommended actions. We call such a distribution a *lottery* and restrict the mediator to rules for which each lottery has a (possibly discrete) density $\sigma(.|\theta)$.

Given a mediation rule $\sigma$ and a recommendation $y$, the receiver’s posterior belief conditional on truthful reporting by the sender is denoted by $\mu(y|\sigma) \in \Delta \Theta$. The mediation rule $\sigma$ is an *equilibrium* one if

$$
y = y^*_z(\mu(y|\sigma)) \text{ for all } y \in \mathcal{A}, \text{ and}
$$

$$
\theta \in \arg \max_{\theta' \in \Theta} \int_{\mathcal{A}} U_S(y, \theta') \sigma(y|\theta') dy \text{ for all } \theta \in \Theta,
$$

where $\mathcal{A} = \bigcup_{\theta' \in \Theta} \sup \sigma(.|\theta')$ and the integral is the interim payoff of a sender of type $\theta$ who reports $\theta'$. Condition (2) is the receiver’s best response (or obedience constraint), and (3) is the sender’s incentive-compatibility constraint.

We say that an action $y$ is *induced* (by an equilibrium $\sigma$) if $y \in \mathcal{A}$, and is *induced by state* $\theta$ if $y \in \sup \sigma(.|\theta)$. To simplify notation when the mediation rule is clear from the

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8By the strict concavity of $U_R(y, \theta)$ in $y$ for all $\theta \in \Theta$, $y^*_z(\mu)$ is unique for any receiver’s posterior belief $\mu \in \Delta \Theta$. Thus, he never mixes over different decisions.

9In the literature, most equilibrium lotteries in models of mediation (or lotteries in the mediated equilibria that are outcome-equivalent to equilibria under other communication protocols) are discrete distributions over recommended actions; e.g., Krishna and Morgan (2004), Blume et al. (2007), Goltsman et al. (2009), Ivanov (2010), Ambrus et al. (2013). For lotteries involving mixtures of continuous and discrete distributions, see Proposition 8 of Blume et al. (2007).
context, the receiver’s interim payoff $EU_R (y' | \mu (y | \sigma))$ from choosing action $y'$ in response to recommendation $y$ is denoted by $EU_R (y' | y)$. The obedience constraint (2) can then be rephrased as:

$$EU_R (y | y) = \max_{y' \in \mathbb{R}} EU_R (y' | y).$$

Note that the uninformative outcome is induced by the mediation rule which recommends the uninformed decision $y_0^1$ for all $\theta \in \Theta$. This rule results in the receiver’s ex-ante payoff

$$EU_R (y_0^1 | y_0^1) = EU_R (y_0^1 | 0, 1).$$

### 3 Beneficial Mediation

Before we proceed to the main results, we introduce a lemma that demonstrates that the supermodularity of payoff functions $U_i (y, \theta), i = S, R$ is preserved if either $y$ or $\theta$ is replaced by distributions ranked by the likelihood ratio order $\succeq_{lr}$. As a result, any equilibrium in the model of mediation is partitional if distributions over actions induced by the mediator can be ranked by the likelihood ratio order. All proofs are collected in the Appendix.

**Lemma 1** Suppose $V (y, \theta)$ is supermodular. If $\sigma (\cdot | \theta_2) \succeq_{lr} \sigma (\cdot | \theta_1)$, then $\Delta V_\sigma (\theta) = E_{\sigma (\cdot | \theta_2)} [V (y, \theta)] - E_{\sigma (\cdot | \theta_1)} [V (y, \theta)]$ is increasing in $\theta$. If $\mu (y_2 | \sigma) \succeq_{lr} \mu (y_1 | \sigma)$, then $\Delta V_\mu (y) = E_{\mu (y_2 | \sigma)} [V (y, \theta)] - E_{\mu (y_1 | \sigma)} [V (y, \theta)]$ is increasing in $y$.

This lemma has two crucial implications. First, it implies that if lotteries over actions are ranked by the likelihood ratio order, then each lottery is induced by an interval of sender’s types. Second, the lemma is important for characterizing the receiver’s optimal decisions given his posterior beliefs about $\theta$. Given $z \in (0, 1)$, denote $F_1 (z) = F_1 (\theta \mid \theta \leq z)$ and $F_2 (z) = F_2 (\theta \mid \theta \geq z)$. Because $F_2 \succeq_{lr} F_1$, then

$$p' F_1 + (1 - p') F_2 \succeq_{lr} p F_1 + (1 - p) F_2$$

for all $0 \leq p' < p \leq 1$.

By Lemma 1, the maximizer (1) is strictly decreasing in $p$ for all $p \in [0, 1]$ and $z \in (0, 1)$. Intuitively, if the receiver’s beliefs put a higher weight on the upper subinterval $[z, 1]$, his optimal decision increases. Then, taking values of $p$ at $1, F (z)$, and $0$ results in

$$y_0^1 < y_0^1 < y_1^1$$

for all $z \in (0, 1)$.

Now, we introduce the separating condition on the fundamentals of the model.

**Condition 1** There exists $z \in (0, 1)$ such that $U_S (y_0^1, z) > \max \{U_S (y_0^1, z), U_S (y_1^1, z)\}$.

Because $U_S (y, \theta)$ is strictly concave in $y$, Condition 1 holds if there exists a sender’s type whose ideal decision coincides with the receiver’s uninformed decision.

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10Consider distributions $G$ and $H$ with (possibly discrete) densities $g$ and $h$, respectively. If $\frac{h (t_1)}{g (t_1)} \geq \frac{h (t_0)}{g (t_0)}$ for all $t_1 > t_0$, where $t_1, t_0 \in \text{supp } G \cup \text{supp } H$, then we say that $H$ dominates $G$ (or $h$ dominates $g$) by the likelihood ratio order (denoted by $H \succeq_{lr} G$ or $h \succeq_{lr} g$). For properties of this order, see Shaked and Shanthikumar (2007).

11The proof follows the same lines as those in Theorem 1.C.30 in Shaked and Shanthikumar (2007).
Condition 2 There exists \( z \in (0, 1) \) such that \( y_S(z) = y_0^1 \).

On one hand, the latter condition is more intuitive and easier to check than Condition 1. On the other hand, Condition 2 is stronger. That is, the existence of \( z = y_S^{-1}(y_0^1) \) is a sufficient, but not necessary for Condition 1.\(^{12}\)

We call a scheme of conflict resolution beneficial if there exists an equilibrium under that scheme which provides a strictly higher ex-ante payoff \( EU_R \) to the receiver compared to the uninformed ex-ante payoff, i.e., \( EU_R > EU_R(y_0^1|y_0^1) \). Given the above preliminaries, we introduce the main result of the paper.

**Proposition 1** Condition 1 is sufficient for beneficial mediation. Also, in any equilibrium of the mediation game with two subintervals and the sender’s cutoff type \( z \in (0, 1) \), Condition 1 holds for \( z \).

The sufficiency part of Proposition 1 is proved by construction. It generalizes the idea used in the literature on mediation for more structured settings by Austen-Smith (1994), Blume et al. (2007) and Ivanov (2010). Intuitively, the uninformed decision \( y_0^1 \) plays the role of the attraction point that allows the mediator to improve the attractiveness of the less beneficial decision in \( \{y_0^1, y_1^1\} \) by partially pooling the sender’s types below and above the cutoff type \( z \). The receiver reacts to this pooling by shifting the less beneficial decision toward \( y_0^1 \) so that type \( z \) becomes indifferent between the two decisions. Thus, if the bias in players’ preferences is sufficiently large but not extreme, the sender’s type space can be separated into two subintervals of types that prefer distinct decisions. For example, suppose that there exists type \( z \in (0, 1) \) such that \( U_S(y_0^1, z) > U_S(y_1^1, z) > U_S(y_0^1, z) \). Without mediation, types \( \theta \leq z \) do not have the incentive to separate themselves because this would induce an unattractive action \( y_1 = y_0^1 \). In order to improve the attractiveness of \( y_1 \) for these types, the mediator partially pools types in \([0, z]\) with those in \([z, 1]\) by randomizing between two recommendations upon receiving signal \( \theta > z \). In response, the receiver increases \( y_1 \) from \( y_0^1 \) toward \( y_1^1 \) so that the cutoff type \( z \) is indifferent between \( y_1 \) and \( y_1^1 \).\(^{13}\) The value of \( y_1 \), however, is limited by the receiver’s obedience constraint \( y_1 \leq y_0^1 \). By the continuity of \( U_S(y, z) \), this constraint holds if \( U_S(y_0^1, z) > U_S(y_1, z) = U_S(y_1^1, z) \), which follows from Condition 1.

The necessity of Condition 1 for two-interval equilibria is proven by contradiction. Suppose that action \( y_1^1 \) is the most attractive among \( y_0^1, y_0^2, \) and \( y_1^1 \) for the cutoff type \( z \). Then, for any lotteries \( \sigma(., \theta_1) \) and \( \sigma(., \theta_2) \) induced by the sender’s types below and above \( z \), respectively, \( \sigma(., \theta_2) \) is strictly preferred by type \( z \), which contradicts the indifference condition for this type.

Condition 1 incorporates two sufficient conditions for beneficial mediation—conditions (ii) and (iii)—in Theorem 1 in Mitusch and Strausz (2005) for the setup with the binary type space and extends them to the case of a continuum of types. In their model, the prior distribution of \( \theta \in \{0, 1\} \) is given by \( \pi = \Pr \{\theta = 1\} \). Denote the

\(^{12}\)Consider the uniform distribution of \( \theta \) and the quadratic preferences with \( y_R(\theta) = \theta \) and \( y_S(\theta) = \min \{1, \theta + 0.39 + \theta \} < \frac{1}{2} = \theta_0^1, \theta \in [0, 1] \). Then, \( U_S(y_0^1, z) > \max \{U_S(y_0^1, z), U_S(y_1^1, z)\} \) for \( z = 0.4 \).

\(^{13}\)By construction, the mediation rule consists of two lotteries, \( \sigma(., \theta_1) \) and \( \sigma(., \theta_2) \), where \( \theta_1 < z < \theta_2 \) and \( \sigma(., \theta_2) \geq \sigma(., \theta_1) \). Hence, by Lemma 1 types below and above \( z \) strictly prefer \( \sigma(., \theta_1) \) and \( \sigma(., \theta_2) \), respectively. That is, conditions (2) and (3) hold.
Similarly, curve type is indifferent between actions equation actions from types above other types. Thus, depend not only on the strategies of the sender’s types that we wish to separate, but also on strategies of functions represents the ideal decision of sender’s type receiver’s posterior belief that \( \theta = 1 \) by \( \mu_1 \), and the receiver’s best response given \( \mu_1 \) by \( y^*_1(\mu_1) \). Then, condition (ii) is \( U_S(y_1^1,1) < U_S(y_0^0,1) \); and (iii) is \( \pi < \pi_2 \), where \( \pi_2 = \arg\max_{\mu_1 \in [0,1]} \{ \mu_1 | U_S(y^*(\mu_1),1) = U_S(y_0^0,1) \} \). Because \( y^*(\mu_1) \) is increasing in \( \mu_1 \), the latter condition results in \( y^*(\pi) = y_0^0 < y^*(\pi_2) \), so that \( U_S(y_1^1,1) > U_S(y_0^0,1) \). Together, (ii) and (iii) lead to \( U_S(y_0^0,y_1^1) > \max \{ U_S(y_0^0,1), U_S(y_1^1,1) \} \), so that Condition 1 holds for type \( z = 1 \). However, because Mitusch and Strausz (2005) define condition (iii) in terms of the prior probability \( \pi \), this condition cannot be extended to non-binary distributions. More importantly, Condition 1 emphasizes that the role of the prior distribution in separating sender’s types is secondary, whereas the primary role is played by the receiver’s response \( y_0^1 \) to the prior information.\(^\text{14}\)

An implication of Proposition 1 is that it allows us to identify scenarios in which mediation facilitates communication while informative direct talk is not feasible.

**Corollary 1** If \( U_S(y_1^1,\theta) \neq U_S(y_0^0,\theta) \) for all \( \theta \in \Theta \) and Condition 1 holds, then: 1) all equilibria in the direct-talk game are uninformative, and 2) mediation is beneficial.

The inequality \( U_S(y_1^1,\theta) \neq U_S(y_0^0,\theta) \) for all \( \theta \in \Theta \) guarantees that only uninformative equilibria exist in the direct-talk game (CS, p. 1440).\(^\text{15}\) Intuitively, this condition implies that there is no type \( z \) indifferent between the receiver’s optimal responses to the information that this type partially separates into two subintervals. Hence, there does not exist an equilibrium with two subintervals. By the monotonicity of equilibria in the number of subintervals, there are no equilibria with a larger number of subintervals.

Fig. 1 illustrates Corollary 1 in the case of the uniform distribution and quadratic payoff functions \( U_i(y,\theta) = -(y - y_i(\theta))^2, i = S, R \), where \( y_R(\theta) = \theta \) is the receiver’s ideal decision. In this figure, curve \( y_A(\theta) = \frac{2+\theta}{4} \) represents the ideal decision of sender’s type \( \theta \in \Theta \), such that this type is indifferent between actions \( y_0^0 = \frac{1}{2} \) and \( y_1^1 = \frac{1+\theta}{2} \). It is implicitly given by the equation \( -(y_1^1 - y)^2 = -(y_0^0 - y)^2 \). It means that \( U_S(y_1^1,\theta) > U_S(y_0^0,\theta) \) if \( y_S(\theta) < y_A(\theta) \).

Similarly, curve \( y_B(\theta) = \frac{1+\theta}{4} \) represents the ideal decision of sender’s type \( \theta \), such that this type is indifferent between actions \( y_0^0 \) and \( y_1^1 \). It is given by the equation \( -(y_0^0 - y)^2 = -(y_1^1 - y)^2 \). Thus, \( U_S(y_1^1,\theta) > U_S(y_0^0,\theta) \) if \( y_S(\theta) > y_B(\theta) \). Finally, curve \( y_C(\theta) = \frac{1+2\theta}{4} \) represents the ideal decision of sender’s type \( \theta \), such that this type is indifferent between actions \( y_0^0 \) and \( y_1^1 \). It is given by the equation \( -(y_0^0 - y)^2 = -(y_1^1 - y)^2 \). Then, \( U_S(y_0^0,\theta) = U_S(y_1^1,\theta) \) if \( y_S(\theta) = y_C(\theta) \). Therefore, there is an informative equilibrium in mediation if \( y_B(z) < y_S(z) < y_A(z) \) for some \( z \in \Theta \) (i.e., \( y_S(z) \) is in the shaded area in the left panel of Fig. 1), whereas there are no informative cheap talk equilibria if \( y_S(\theta) \neq y_C(\theta) \) for all \( \theta \in \Theta \).

\(^\text{14}\) Also, in contrast to the binary case, the continuity of the type space means that the receiver’s decisions depend not only on the strategies of the sender’s types that we wish to separate, but also on strategies of other types. Thus, \( y_0^0 \) and \( y_1^1 \) implicitly assume that sender’s types below \( z \) prefer to separate themselves from types above \( z \) and vice versa.

\(^\text{15}\) Corollary 1 in CS, which establishes that there exist only uninformative cheap-talk equilibria if \( U_S(y_1^1,\theta) \neq U_S(y_0^0,\theta) \) for all \( \theta \in \Theta \), stems from their Theorem 1. This theorem also assumes that \( y_S(\theta) \neq y_R(\theta) \) for all \( \theta \in \Theta \). In contrast, our setup does not impose such a restriction. However, as noted by CS (p. 1437), relaxing the condition “\( y_S(\theta) \neq y_R(\theta) \) for all \( \theta \in \Theta \)” does not affect the conditions under which all cheap-talk equilibria are uninformative, but may result in equilibria with an infinite number of subintervals in some cases. Gordon (2010) investigates situations in which equilibria contain an infinite number of subintervals.
int$\Theta$ (see the right panel of Fig. 1). Consider, for instance $y_S(\theta) = \min \{ \theta + \varepsilon, \frac{\theta + 1 - \varepsilon}{2} \}$. In this case, even though $y_S(\theta)$ is arbitrarily close to $y_R(\theta)$ for an interval of states as $\varepsilon \to 0$, cheap talk is always uninformative if $\varepsilon > 0$. In contrast, mediation is beneficial if $0 < \varepsilon < 1$.

4 Comparison to other schemes of conflict resolution

In this section, we compare our condition with the related conditions in alternative schemes of conflict resolution, in particular, delegation (Alonso and Matouschek, 2008), mediation via a biased mediator (Ambrus et al., 2013), and arbitration (Kovác and Mylovanov, 2009). The main implication of this comparison is identifying the set of scenarios in which multiple schemes of conflict resolution can be beneficial. Following these authors, we consider the generalized quadratic payoff functions:

$$U_i(y, \theta) = -(y - y_i(\theta))^2, \ i = S, R.$$  

**Delegation.** According to Alonso and Matouschek (2008), delegation is beneficial if and only if the players’ preferences are *minimally aligned*, that is,

$$y_0^z < y_S(z) < y_1^z \text{ for some } z \in (0, 1).$$  

(4)

**Mediation via a biased mediator.** Ambrus et al. (2013) provide two conditions, which are together sufficient for beneficial mediation via a biased mediator:

$$U_S(y_0^0, 0) < U_S(y_0^1, 0) \text{ and } y_S(0) < y_0^1.$$  

(5)

**Arbitration.** Kovác and Mylovanov (2009) assign $y_S(\theta) = \theta$ and impose the following
regularity condition on \( F(\theta) \) and \( y_R(\theta) \).\(^\text{16}\)

**Condition 3** If \( g(\theta) \in [0, 1] \), then \( g(\theta) \) is decreasing at \( \theta \), where \( g(\theta) = 1 - F(\theta) + (y_R(\theta) - \theta) f(\theta) \).

According to Kovác and Mylovanov (2009), arbitration is beneficial if and only if

\[
0 \leq \alpha_0 < \beta_0 \leq 1, \quad (6)
\]

where \( \alpha_0 = \max \{ \theta \in [0, 1] | y_0^\theta \geq y_S(\theta) \} \), and \( \beta_0 = \min \{ \theta \in [0, 1] | y_0^1 \leq y_S(\theta) \} \).

The following proposition demonstrates the relationship between the aforementioned conditions and Condition 1.

**Proposition 2** Condition 1 is strictly stronger than (4) and strictly weaker than (5). Also, if Condition 3 holds, then Condition 1 is equivalent to (6).

Intuitively, Condition 1 is similar to (4) with additional constraints stemming from the receiver’s obedience in mediation. In delegation, condition (4) implies that the state space can be separated into subintervals via commitment to a pair of actions (contingent on the sender’s message) in such a way that the ex-ante payoff to the receiver increases. For example, if \( U_S(y_1^1, z) > U_S(y_0^\theta, z) \), then the receiver can commit to action \( y_1 \) upon receiving message \( \theta \leq z \), where \( y_1 < y_S(\theta) \) is such that \( U_S(y_1, z) = U_S(y_1^1, z) > U_S(y_0^\theta, z) \). Commitment, however, does not require the optimality of \( y_1 \) for the receiver given sender’s message that \( \theta < z \).\(^\text{17}\) Due to this, the receiver’s obedience constraint in mediated communication, \( y_1 \leq y_0^1 \), is not necessary in delegation. As a result, even though (4) is satisfied, the inequality \( U_S(y_0^1, z) > U_S(y_1^1, z) \), and hence, Condition 1, may not hold.

Also, Condition 1 generalizes local conditions (5) for the lowest type \( \theta_0 = 0 \). By continuity, (5) implies that Condition 1 holds for sender’s types close to \( \theta_0 \). On the other hand, Condition 1 states that the possibilities for beneficial mediation do not depend on the incentives of extreme sender’s types or on the ranking of the sender’s ideal decision \( y_S(z) \) and the receiver uninformed action \( y_0^1 \).

Finally, (6) and Condition 3 imply that there exists \( z \) such that \( y_S(z) = y_0^1 \), so that Condition 2 holds. Hence, Condition 1 holds also. Conversely, if Condition 1 is satisfied, then mediation is beneficial by Proposition 1. Because any mediated equilibrium can be induced in arbitration, then arbitration must be beneficial also. At the same time, Kovác and Mylovanov (2009) show that under Condition 3, arbitration is beneficial if and only if (6) holds.

## 5 Conclusion and Discussion

Though it has been known that introducing a mediator in cheap talk between conflicting parties can facilitate communication, the set of scenarios in which the receiver can elicit

\(^{16}\)Assigning \( y_S(\theta) = \theta \) is without loss of generality. Because \( y_S(\theta) \neq \theta \) is strictly increasing and continuous, then the transformation of state \( \theta' = y_S(\theta) \) results in \( V(y, \theta') = - (y - \theta')^2 \).

\(^{17}\)If \( y_1 > y_0^1 \) in delegation, then decision \( y_1 \) is less optimal for the receiver than \( y_0^1 \) if \( \theta < z \). However, his ex-ante payoff increases compared to that in the uninformative equilibrium due to the optimality of decision \( y_1^1 \) if \( \theta \geq z \).
meaningful information from the sender has not been explored in the general CS framework. This work fills this gap by identifying the sufficient condition under which the mediator is beneficial to the uninformed decision maker. We also investigate the necessity of this condition and show how it is related to conditions for beneficial interaction of parties with conflicting interests in other schemes of conflict resolutions.

It is important to note that though Condition 1 is sufficient for facilitating communication over the uninformative outcome in cheap talk, it is not sufficient for improving informative cheap-talk outcomes.\(^\text{18}\) Intuitively, the lotteries (in mediation) which elicit more information from some sender’s types also distort the incentives of other types. As a result, these types may (stochastically) induce actions which are less efficient for the receiver compared to those induced in the original cheap-talk equilibrium. In general, the possibility of the mediator to improve on the informative cheap-talk equilibria hinges on the specifications of the sender-receiver model, in particular, on the relationship between the conflict in players’ preferences and the prior distribution of states. In recent work, Alonso and Rantakari (2013) characterize a class of settings in which the receiver’s ex-ante payoff in the most informative cheap-talk equilibrium cannot be strictly improved by mediation. It is worth noting that our condition holds in this class. Because of these arguments, the characterization of sufficient conditions under which mediation strictly improves informative cheap-talk equilibria remains an open question.

**Appendix**

**Proof of Lemma 1.** Consider two lotteries \(\sigma(\cdot|\theta_2) \succeq_{LR} \sigma(\cdot|\theta_1)\), and let \(\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2\), where \(\mathcal{A}_i = \text{supp} \sigma(\cdot|\theta_i)\), \(i = 1, 2\). (Hereafter, we present the argument for densities; the proof for discrete distributions proceeds along the same line.) Then, \(\mathcal{A}_2 \) dominates \(\mathcal{A}_1\) by the strong set order and \(\frac{\sigma(y|\theta_2)}{\sigma(y|\theta_1)}\) is increasing in \(y\). Consider \(H: \mathcal{A} \to \mathbb{R}\), such that

\[
H(y) = \sigma(y|\theta_2) - \sigma(y|\theta_1) = \begin{cases} 
\frac{\sigma(y|\theta_1)\left(\frac{\sigma(y|\theta_2)}{\sigma(y|\theta_1)} - 1\right)}{\sigma(y|\theta_2)} & \text{if } \sigma(y|\theta_1) > 0, \\
\sigma(y|\theta_2) & \text{if } \sigma(y|\theta_1) = 0. 
\end{cases}
\] (7)

Note that \(\int_{\mathcal{A}} H(y) \, dy = 0\). Because \(\frac{\sigma(y|\theta_2)}{\sigma(y|\theta_1)}\) is increasing in \(y\), \(H(y)\) is quasi-monotone in \(y\), i.e., there exists \(y_c \in \mathcal{A}_1 \cap \mathcal{A}_2\), such that \(H(y) \leq 0\) if \(y \leq y_c\), and \(H(y) \geq 0\) if \(y \geq y_c\).

\(^{18}\text{Consider, for example, the uniform-quadratic setup with } b = \frac{1}{5}. \text{ In this case, the optimal equilibrium in mediated communication coincides with the most informative cheap-talk equilibrium. This equilibrium involves two intervals with cutoff } \theta_1 = \frac{1}{3}, \text{ and actions } a_1 = E[\theta|\theta \leq \theta_1] = \frac{1}{5} \text{ and } a_2 = E[\theta|\theta \geq \theta_1] = \frac{3}{5}. \text{ However, Condition 2 holds for } z = \frac{1}{2} \text{ on interval } [\theta_1, 1], \text{ since } y_S(\frac{1}{2}) = y_{S_1} = \frac{3}{5}. \text{ Thus, it is possible to facilitate communication about states in } [\theta_1, 1] \text{ by separating this interval into two subintervals. In particular, there exists an equilibrium with cutoff types } \{\theta'_1, z\} = \left\{\frac{1}{6}, \frac{1}{2}\right\}, \text{ and actions } \{a'_1, a'_2, a'_3\} = \left\{\frac{1}{5}, \frac{1}{2}, \frac{3}{5}\right\}. \text{ In this equilibrium, the mediator purely induces actions } a'_1, a'_2 \text{ if } \theta < \theta'_1 \text{ and } \theta \in [\theta'_1, z], \text{ respectively, and mixes between } a'_2 \text{ and } a'_3 \text{ with probabilities } \frac{1}{3} \text{ and } \frac{2}{3}, \text{ respectively, if } \theta > z. \text{ However, the ex-ante payoff of the receiver is } -\frac{1}{27}, \text{ which is lower than that of } -\frac{1}{192} \text{ in the original equilibrium.}
Consider $\theta_2 > \theta_1$ and

$$\Delta V_\sigma (\theta_2) - \Delta V_\sigma (\theta_1) = \int_A (V(y, \theta_2) - V(y, \theta_1)) H(y) \, dy.$$  

Because $V(y, \theta_2) - V(y, \theta_1)$ is increasing in $y$ by the supermodularity of $V(y, \theta)$, then by Lemma 1 in Persico (2000), we have $\Delta V_\sigma (\theta_2) - \Delta V_\sigma (\theta_1) \geq 0$.

Similarly, if $\mu(y_2|\sigma) \succeq_{tr} \mu(y_1|\sigma)$, then putting $G = \mu(y_2|\sigma) - \mu(y_1|\sigma)$ and using the same argument as above implies

$$\Delta V_\mu (y_2) - \Delta V_\mu (y_1) = \int (V(y_2, \theta) - V(y_1, \theta)) G(\theta) \, d\theta \geq 0 \text{ if } y_2 > y_1.$$

Before proceeding to other results, we prove the following claim.

**Claim 1** If sender’s strategy is such that $\theta$ below and above $z \in (0, 1)$ induce lotteries $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$, respectively, then $\sigma(.|\theta_2) \succeq_{tr} \sigma(.|\theta_1)$.

**Proof** Suppose that the mediation rule consists of two lotteries, such that sender’s types below and above $z \in (0, 1)$ induce lotteries $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$, respectively. By the obedience constraint (2), we have

$$y = y^*_z(p(y, z)), y \in A,$$

where

$$p(y, z) = \frac{F(z) \sigma(y|\theta_1)}{F(z) \sigma(y|\theta_1) + (1 - F(z)) \sigma(y|\theta_2)} = \frac{1}{1 + \varphi(y, z)},$$

is the posterior probability that $\theta \in [0, z]$ conditional on message $y$, and

$$\varphi(y, z) = \frac{(1 - F(z)) \sigma(y|\theta_2)}{F(z) \sigma(y|\theta_1)},$$

is the likelihood ratio $\frac{\sigma(y|\theta_2)}{\sigma(y|\theta_1)}$ of inducing action $y$ adjusted to the prior distribution of $\theta$.

Since $y^*_z(p)$ is strictly decreasing in $p$, then (8) implies that $p(y, z)$ is strictly decreasing in $y$. Then, (9) implies that $\varphi(y, z)$ is strictly increasing in $y$. Thus, $\frac{\sigma(y|\theta_2)}{\sigma(y|\theta_1)}$ is strictly increasing in $y$ for all $y \in \text{supp } \sigma(.|\theta_1) \cup \text{supp } \sigma(.|\theta_2)$, i.e., $\sigma(.|\theta_2) \succeq_{tr} \sigma(.|\theta_1)$. Also, if $y^*_1 \in A$, then $\sigma(y^*_1|\theta_1) = \sigma(y^*_1|\theta_2)$ and $p(y^*_1, z) = F(z)$. ■

**Proof of Proposition 1. (Sufficiency)** Suppose that there is $z \in (0, 1)$ such that $U_S(y^*_0, z) > \max \{U_S(y^*_0, z), U_S(y^*_1, z)\}$ and consider two cases.

(A) Let $U_S(y^*_1, z) \geq U_S(y^*_0, z)$. Because $U_S(y^*_0, z) > U_S(y^*_1, z)$ and $y^*_0 < y^*_1 < y^*_1$, there exists a unique $y_1 \in [y^*_0, y^*_1)$, such that $U_S(y_1, z) = U_S(y^*_1, z)$. Consider the mediation rule that consists of two lotteries. Lottery $\sigma(.|\theta_1)$ purely maps the sender’s reported types $\theta < z$ into action $y_1$. If $\theta \geq z$, lottery $\sigma(.|\theta_2)$ randomizes between actions $y_1$ and $y^*_1$ with

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19Claim 1 does not hold if the number of subintervals exceeds two (for a counterexample, see Blume et al., 2007).
probabilities $\sigma(y_1|\theta_2)$ and $1 - \sigma(y_1|\theta_2)$, respectively. Given the mediation rule $\sigma$, the posterior probability that $\theta \in [0, z]$ conditional on recommended action $y_1$ is

$$p(y_1, z) = \frac{F(z)}{F(z) + (1 - F(z)) \sigma(y_1|\theta_2)}.$$ 

Since $y_1 \in [y_0^*, y_0^0)$, and the receiver’s best-response $y^*_z(p)$ is such that $y^*_z(F(z)) = y_0^1, y_0^2(1) = y_0^0$, and $y^*_z(p)$ is strictly decreasing and continuous in $p$, there is a unique posterior belief $p^* \in (F(z), 1)$, such that $y^*_z(p^*) = y_1$. Because $p(y_1, z) \in [F(z), 1]$ if $\sigma(y_1|\theta_2) \in [0, 1]$, and $p(y_1, z)$ is strictly decreasing in $\sigma(y_1|\theta_2)$, there is a unique $\sigma^* \in [0, 1)$, such that $p(y_1, z) = p^*$ for $\sigma(y_1|\theta_2) = \sigma^*$.

In addition, $U_2(y_1, z) = U_2(y_2^1, z)$ means that type $z$ is indifferent between $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$, where $\sigma(.|\theta_2) \succ_{tr} \sigma(.|\theta_1)$ by construction. By Lemma 1, types $\theta$ below and above $z$ strictly prefer $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$, respectively. Also, $y_2^1$ is induced by $\theta > z$ only, i.e., it satisfies (2).

Finally, note that $p^* > 0$ and $z \in (0, 1)$ imply $y_0^0 \notin \{y_1, y_2^1\}$. By the uniqueness of $y^*_z(p)$, we have $U_R(y|y) > U_R(y_0^1|y)$ if $y \neq y_0^0$, so that the receiver’s ex-ante payoff is

$$EU_R = \Pr \{y_1\} EU_R(y_1|y) + \Pr \{y_2^1\} EU_R(y_2^1|y_1)$$

$$> \Pr \{y_1\} EU_R(y_0^1|y_1) + \Pr \{y_2^1\} EU_R(y_0^1|y_2^1) = EU_R(y_0^1|y_0^0).$$

(B) If $U_S(y_2^1, z) < U_S(y_0^0, z)$, the proof is similar. The only difference from (A) is the mediation rule. In particular, lottery $\sigma(.|\theta_1)$ randomizes between recommending actions $y_0^z$ and $y_2$ with probabilities $1 - \sigma(y_2|\theta_1)$ and $\sigma(y_2|\theta_1)$ for reported types $\theta \leq z$. Also, lottery $\sigma(y_2|\theta_2)$ purely maps $\theta > z$ into action $y_2 = y_2^z(p(y_2, z))$, where

$$p(y_2, z) = \frac{F(z) \sigma(y_2|\theta_1)}{F(z) \sigma(y_2|\theta_1) + 1 - F(z)}.$$ 

(Necessity) Consider an equilibrium with two subintervals, such that sender’s types $\theta$ below and above $z \in (0, 1)$ induce lotteries $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$, respectively. By Claim 1, $\sigma(.|\theta_2) \succ_{tr} \sigma(.|\theta_1)$. First, we show that

$$y_0^z \leq \inf A < y_S(z) < \sup A \leq y_2^1. \tag{10}$$

By contradiction, suppose that $\sup A \leq y_S(z)$. Because $U_S(y, z)$ is strictly increasing in $y$ for $y \leq y_S(z)$, it follows that the difference in the sender’s interim payoffs from inducing $\sigma(.|\theta_2)$ and $\sigma(.|\theta_1)$ is

$$\Delta U_S(z) = \int_A U_S(y, z) \sigma(y|\theta_2) dy - \int_A U_S(y, z) \sigma(y|\theta_1) dy > 0,$$

since the likelihood-ratio order implies the first-order stochastic order. This, however, contradicts the indifference of type $z$ between $\sigma(.|\theta_1)$ and $\sigma(.|\theta_2)$. Thus, $\sup A < y_S(z)$.

By the same argument, $\inf A < y_S(z)$. In addition, $y_0^0 = y_0^z(1), y_2^1 = y_2^z(0)$, and the monotonicity of $y^*_z(p)$ in $p$ imply $y \in [y_0^z, y_2^z]$, which gives (10).
Suppose by contradiction that $U_S(y^*_2, z) \geq U_S(y^*_1, z) > U_S(y^*_0, z)$. This implies $y^*_0 < y^*_S(z)$. (Otherwise, if $y^*_0 \geq y^*_S(z)$, then the fact that $U_S(y, z)$ is strictly decreasing in $y$ for $y \geq y^*_S(z)$, and $y^*_0 < y^*_1$ result in the contradiction $U_S(y^*_1, z) < U_S(y^*_0, z)$). Also, $U_S(y, z) < U_S(y^*_0, z)$ if $y \leq y^*_0$, since $y^*_0 < y^*_S(z)$ and $U_S(y, z)$ is strictly increasing in $y$ for $y \leq y^*_S(z)$.

Since $\sigma(y^*_0(\theta))$ is strictly increasing in $y$, then $H(y)$ defined by (7) is strictly quasi-monotone. Also, $\sigma(y^*_0(\theta)) = \sigma(y^*_0(\theta_2))$ implies $H(y^*_0) = 0$, i.e., $H(y) < 0$ if $y < y^*_0$, and $H(y) > 0$ if $y > y^*_0$. Consider

$$\tilde{U}_S(y, z) = U_S(y, z) - U_S(y^*_0, z).$$

Then,

$$\Delta U_S(z) = \int_A H(y) U_S(y, z) \, dy = \int_A H(y) \tilde{U}_S(y, z) \, dy$$

$$= \int_{\{y \in A \mid y \leq y^*_0\}} H(y) \tilde{U}_S(y, z) \, dy + \int_{\{y \in A \mid y > y^*_0\}} H(y) \tilde{U}_S(y, z) \, dy,$$

where the first equality is due to $\int_A H(y) \, dy = 0$. Since $H(y) < 0$ and $\tilde{U}_S(y, z) < 0, y < y^*_0$, it follows that $\int_{\{y \in A \mid y > y^*_0\}} H(y) \tilde{U}_S(y, z) \, dy > 0$. Also, the strict concavity of $U_S(y, z)$ in $y$ implies

$$\tilde{U}_S(y, z) > \min \left\{ \tilde{U}_S(y^*_0, z), \tilde{U}_S(y^*_1, z) \right\} = \tilde{U}_S(y^*_0, z) = 0, y \in (y^*_0, y^*_1],$$

and

$$\int_{\{y \in A \mid y > y^*_0\}} H(y) \tilde{U}_S(y^*_0, z) \, dy = 0.$$

This leads to $\Delta U_S(z) > 0$, which contradicts the indifference condition $\Delta U_S(z) = 0$. The case of $U_S(y^*_0, z) \geq U_S(y^*_1, z) > U_S(y^*_2, z)$ is proved similarly.

**Proof of Proposition 2.** (a) By contradiction, suppose Condition 1 holds for some $z \in (0, 1)$ and $y^*_S(z) \geq y^*_1$. Because $U_S(y, z)$ is strictly increasing in $y$ for $y \leq y^*_S(z)$, then $y^*_1 < y^*_S(z)$ leads to $U_S(y^*_0, z) < U_S(y^*_1, z)$, which is a contradiction to $U_S(y^*_1, z) > U_S(y^*_2, z)$. Similarly, if we assume that $y^*_S(z) \leq y^*_0$, then the fact that $U_S(y, z)$ is strictly decreasing in $y$ for $y \geq y^*_S(z)$ and $y^*_0 < y^*_S(z)$ imply that $U_S(y^*_1, z) < U_S(y^*_0, z)$. This contradicts $U_S(y^*_0, z) > U_S(y^*_1, z)$. Thus, $y^*_S(z) \in (y^*_0, y^*_1)$.

In order to show that the converse does not hold, consider the uniform-quadratic setup with $y_R(\theta) = \theta$ and $y_S(\theta) = \frac{\theta^2}{2} (1 + \theta)$. This implies $y^*_0 = \frac{\theta^2}{2}$ and $y^*_0 = \frac{1+\theta}{2} \theta$. Then, $y^*_S(\theta) \in (y^*_0, y^*_1)$ if and only if $\theta \in (0, \frac{1}{4})$. However, $U_S(y^*_0, \theta) - U_S(y^*_0, \theta) = -\frac{5\theta^2 - 4\theta + 1}{4} < 0$ for $\theta \in (0, \frac{1}{4})$.

(b) Suppose (5) holds. Since $U_S(y^*_0, 0) < U_S(y^*_1, 0)$, then by continuity of $U_S(y, \theta)$ in $(y, \theta)$, and $y^*_0$ and $y^*_1$ in $z$, it follows that $U_S(y^*_0, z) < U_S(y^*_1, z)$ for $z \in (0, \epsilon_1)$, where $\epsilon_1 > 0$. Also, $y^*_S(0) < y^*_0 < y^*_1, z > 0$ means that $y^*_S(z) < y^*_0 < y^*_1$ if $z \in (0, \epsilon_2)$, where $\epsilon_2 > 0$. Since $U_S(y, z)$ is strictly decreasing in $y$ if $y \geq y^*_S(z)$, this implies $U_S(y^*_0, z) > U_S(y^*_1, z) > U_S(y^*_0, z)$, $z \in (0, \min(\epsilon_1, \epsilon_2))$. Thus, Condition 1 holds.

In order to show that Condition 1 does not imply either of conditions in (5), consider
the uniform-quadratic setup with \( y_R (\theta) = \theta \). First, put \( y_S (\theta) = \max \left\{ \frac{5}{9}, \theta + \frac{1}{18} \right\} \). Then, \( y_S (0) = \frac{5}{9} > \frac{1}{2} = y_0^1 \). However, for \( z = \frac{1}{2} \), we have \( y_0^1 = \frac{1}{4} \), \( y_z^1 = \frac{3}{4} \), \( y_S (z) = \frac{5}{9} \). Since

\[
|y_0^1 - y_S (z)| = \frac{1}{18} < \frac{7}{36} = \min \left\{ |y_0^1 - y_S (z)|, |y_z^1 - y_S (z)| \right\},
\]

this means Condition 1 holds.

Also, put \( y_S (\theta) = \theta + b \), where \( b \in (0, \frac{1}{4}) \). For \( z = \frac{1}{2} - b \), we have \( y_S (z) = y_0^1 \), that is, Condition 2 holds. On the other hand, \( |y_0^1 - y_S (0)| = b < \frac{1}{2} - b = |y_1^1 - y_S (0)| \) implies \( U_S (y_0^0, 0) > U_S (y_0^1, 0) \).

(c) Suppose (6) holds. Then, \( y_0^1 > y_S (0) = 0 \). Otherwise, \( y_0^1 \leq y_S (0) \) implies \( \beta_0 = 0 \), which results in the contradiction \( 0 \leq \alpha_0 < \beta_0 = 0 \). Similarly, \( y_0^1 < 1 = y_S (1) \). Otherwise, \( y_0^1 \geq y_S (1) \) implies \( \alpha_0 = 1 \), which results in the contradiction \( 1 = \alpha_0 < \beta_0 \leq 1 \). Because \( y_S (0) < y_0^1 < y_S (1) \) and \( y_S (\theta) \) is continuous in \( \theta \), there exists \( z \), such that \( y_S (z) = y_0^1 \). This implies that Condition 1 holds.

Conversely, if Condition 1 holds, then mediation is beneficial. Because the set of equilibria in mediation is a subset of the set of equilibria in arbitration with the obedience constraint (2), arbitration is also beneficial. However, Kovác and Mylovanov (2009) show that under Condition 3 arbitration is beneficial if and only if (6) holds. ■

References


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Formally, \( y_S (\theta) = \frac{5}{9} \) if \( \theta < \frac{1}{7} \). Thus, \( y_S (\theta) \) is not strictly increasing in \( \theta \) as required by the strict supermodularity of \( U_S (y, \theta) \). However, an approximation of \( y_S (\theta) \) by \( \tilde{y}_S (\theta) \) is \( \max \left\{ \frac{5}{9} + \varepsilon \theta, \theta + \frac{1}{18} \right\} \), where \( \varepsilon > 0 \) does not affect the result.

Note that in both examples above, we have \( b (\theta) = y_S (\theta) - y_R (\theta) > 0 \), so that condition \( b (\theta) > 0 \) for all \( \theta \in \Theta \) in Proposition 6 in Ambrus et al. (2013) is satisfied.


