THE THEORY

OF

POLITICAL ECONOMY

BY

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WITH PREFACE AND NOTES AND AN EXTENSION OF THE BIBLIOGRAPHY OF MATHEMATICAL ECONOMIC WRITINGS

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FIFTH EDITION

First Edition, 1871.
Second Edition, 1879.
Third Edition, 1888.
Fourth Edition, 1911.

PREFACE TO THE FIFTH EDITION

In writing the Preface to the fourth edition of the *Theory of Political Economy* I ventured to predict that it would be the last. That edition was in fact exhausted a few years ago; but Messrs. Macmillan were unable to see their way to reprint the book. I was glad therefore to have an offer from the representative of an American firm to bring out a new edition, for there is a continuing demand for the *Theory*, which has found a place amongst the economic classics of the nineteenth century.

Its appeal lies not merely in the Author's clarity and freshness of diction, but in its combination of two previously separate schools of thought — those favoring respectively the psychological and the mathematical methods in economics. It is still a matter of controversy whether economics should be regarded as a calculus of pleasure and pain or, more correctly, of positive and negative feeling, or whether it should be treated as the science of preferences in the satisfaction of human wants. For myself, I believe that those who refuse or neglect to study the psychological basis of economics as one branch of the science of human behavior are less likely than those who do so to arrive at sound conclusions

tending in their applications to maximise human happiness.

In the preface to the fourth edition I explained that it had been my intention to continue my father's bibliography of mathematico-economic writings by drawing up a bibliography of all books and articles of that kind published from 1880 to 1910; but that as the work was still very far from complete I had decided that the new edition should be issued without it. It was my intention that it should be published later as an independent volume.

Unfortunately, no sooner was the fourth edition published than I became deeply interested in housing reform in the mining valleys of South Wales, and in a few months had resigned from the chair of Economics and Political Science at Cardiff to devote my whole time to housing reform. In 1914, soon after the outbreak of war, I was appointed to the newly established research chair in economics in the University of Allahabad. From 1911 on, therefore, only spasmodic progress was made with the bibliography; and finally I gave up all hope of completing it, though I still have most of the cards.

The purpose of the intended bibliography has been partially served by the publication of *A Select Bibliography of Modern Economic Theory*, 1870 to 1929, edited by H. E. Batson, in the series of bibliographies issued by the London School of Economics. Mention may also be made of the bibliography of mathematico-economic writings prepared by Professor Irving Fisher and published first as an appendix to his article entitled *Cournot and Mathematical Economics* (*Quarterly Journal of Economics, January*, 1898), and later in his edi-

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tion of Cournot's *Researches into the Mathematical Principles of the Theory of Wealth* (Economic Classics Series, 1927). Professor Fisher's bibliography is classified and annotated; and is probably an almost complete list of all published writings on economics of a mathematical character up to 1897. He did not add entries for the later years when reprinting the bibliography in 1927, for, as he wrote in the preface, the mathematical method was now so widely recognised as applicable to economics that the number of publications to be included in the bibliography would have been greatly increased and there was also less need to emphasise its importance.

In preparing the third edition of the *Theory*, my mother, with the assistance of Mr. Foxwell and others, continued the bibliography to 1888 and added a few titles which my father had not discovered. These latter are shown in square brackets. When editing the fourth edition I omitted all titles dated after 1879 as it was my intention to include them in the separate bibliography mentioned above. As this omission may seem to have detracted somewhat from the value of the fourth edition, the part of the bibliography omitted from it is now restored, so that it appears exactly as in the third edition, except for the correction of a few printer's errors.

I close this preface with the hope that looking back to the development of the marginal theory in the nineteenth century will prove as stimulating to students of economics in the United States as in Britain, now that the *Theory of Political Economy* is once more available. I shall not venture to prophesy again as to whether this will be the last edition.

PREFACE TO THE FIRST EDITION

(1871)

THE contents of the following pages can hardly meet with ready acceptance among those who regard the Science of Political Economy as having already acquired a nearly perfect form. I believe it is generally supposed that Adam Smith laid the foundations of this science; that Malthus, Anderson, and Senior added important doctrines; that Ricardo systematised the whole; and, finally, that Mr. J. S. Mill filled in the details and completely expounded this branch of knowledge. Mr. Mill appears to have had a similar notion; for he distinctly asserts that there was nothing in the Laws of Value which remained for himself or any future writer to clear up. Doubtless it is difficult to help feeling that opinions adopted and confirmed by such eminent men have much weight of probability in their favour. Yet, in the other sciences this weight of authority has not been allowed to restrict the free examination of new opinions and theories; and it has often been

ultimately proved that authority was on the wrong side.

There are many portions of Economical doctrine which appear to me as scientific in form as they are consonant with facts. I would especially mention the Theories of Population and Rent, the latter a theory of a distinctly mathematical character, which seems to give a clue to the correct mode of treating the whole science. Had Mr. Mill contented himself with asserting the unquestionable truth of the Laws of Supply and Demand, I should have agreed with him. As founded upon facts, those laws cannot be shaken by any theory; but it does not therefore follow that our conception of Value is perfect and final. Other generally accepted doctrines have always appeared to me purely delusive, especially the so-called Wage Fund Theory. This theory pretends to give a solution of the main problem of the science—to determine the wages of labour; yet, on close examination, its conclusion is found to be a mere truism, namely, that the average rate of wages is found by dividing the whole amount appropriated to the payment of wages by the number of those between whom it is divided. Some other supposed conclusions of the science are of a less harmless character, as, for instance, those regarding the advantage of exchange (see the section on "The Gain by Exchange," p. 142).

In this work I have attempted to treat Economy as a Calculus of Pleasure and Pain, and have

sketched out, almost irrespective of previous opinions, the form which the science, as it seems to me, must ultimately take. I have long thought that as it deals throughout with quantities, it must be a mathematical science in matter if not in language. I have endeavoured to arrive at accurate quantitative notions concerning Utility, Value, Labour, Capital, etc., and I have often been surprised to find how clearly some of the most difficult notions, especially that most puzzling of notions *Value*, admit of mathematical analysis and expression. The Theory of Economy thus treated presents a close analogy to the science of Statical Mechanics, and the Laws of Exchange are found to resemble the Laws of Equilibrium of a lever as determined by the principle of virtual velocities. The nature of Wealth and Value is explained by the consideration of indefinitely small amounts of pleasure and pain, just as the Theory of Statics is made to rest upon the equality of indefinitely small amounts of energy. But I believe that dynamical branches of the Science of Economy may remain to be developed, on the consideration of which I have not at all entered.

Mathematical readers may perhaps think that I have explained some elementary notions, that of the Degree of Utility for instance, with unnecessary prolixity. But it is to the neglect of Economists to obtain clear and accurate notions of quantity and degree of utility that I venture to attribute the present difficulties and imperfections of the science;

and I have purposely dwelt upon the point at full length. Other readers will perhaps think that the occasional introduction of mathematical symbols obscures instead of illustrating the subject. But I must request all readers to remember that, as Mathematicians and Political Economists have hitherto been two nearly distinct classes of persons, there is no slight difficulty in preparing a mathematical work on Economy with which both classes of readers may not have some grounds of complaint.

It is very likely that I have fallen into errors of more or less importance, which I shall be glad to have pointed out; and I may say that the cardinal difficulty of the whole theory is alluded to in the section of Chapter IV. upon the "Ratio of Exchange," beginning at p. 91 (that on "the Law of Indifference," p. 90 of this edition). So able a mathematician as my friend Professor Barker, of Owens College, has had the kindness to examine some of the proof sheets carefully; but he is not, therefore, to be held responsible for the correctness of any part of the work.

My enumeration of the previous attempts to apply mathematical language to Political Economy does not pretend to completeness even as regards English writers; and I find that I forgot to mention a remarkable pamphlet "On Currency" published anonymously in 1840 (London, Charles Knight and Co.) in which a mathematical analysis of the operations of the Money Market is attempted. The

method of treatment is not unlike that adopted by Dr. Whewell, to whose Memoirs a reference is made; but finite or occasionally infinitesimal differences are introduced. On the success of this anonymous theory I have not formed an opinion; but the subject is one which must some day be solved by mathematical analysis. Garnier, in his treatise on Political Economy, mentions several continental mathematicians who have written on the subject of Political Economy; but I have not been able to discover even the titles of their Memoirs.

PREFACE TO THE SECOND EDITION

(1879)

In preparing this second edition certain new sections have been added, the most important of which are those treating of the *dimensions of economic quantities* (pp. 61-69, 83-84, 178-179, 233-235). The subject, of course, is one which lies at the basis of all clear thought about economic science. It cannot be surprising that many debates end in logomachy, when it is still uncertain how many meanings the word *value* has, or what kind of a quantity *utility* itself is. Imagine the mental state of astronomers if they could not agree among themselves whether *Right Ascension* was the name of a heavenly body, or a force or an angular magnitude. Yet this would not be worse than failing to ascertain clearly whether by value we mean a numerical ratio, or a mental state, or a mass of commodity. John Stuart Mill tells us explicitly[1] that "The value of a thing means the quantity of some other thing, or

[1] *Principles of Political Economy*, book iii. chap. vi. sec. i. 1. This definition occurs at the beginning of a carefully prepared summary of the principles of the theory of value.

of things in general, which it exchanges for." It might of course be explained that Mill did not intend what he said; but as the statement stands it makes value into a thing, and is just as philosophic as if one were to say, "Right Ascension means the planet Mars, or planets in general."

These sections upon the dimensions of economic quantities have caused me great perplexity, especially as regards the relation between utility and time (pp. 64-69). The theory of capital and interest also involves some subtleties. I hope that my solutions of the questions raised will be found generally correct; but where they do not settle a question, they may sometimes suggest one which other writers may answer. A correspondent, Captain Charles Christie, R.E., to whom I have shown these sections after they were printed, objects reasonably enough that commodity should not have been represented by M, or Mass, but by some symbol, for instance Q, which would include quantity of space or time or force, in fact almost any kind of quantity. Services often involve time, or force exerted, or space passed over, as well as mass. In this objection I quite concur, and I must therefore request the reader either to interpret M with a wider meaning than is given to it in p. 64, or else mentally to substitute another symbol.

In treating the dimensions of interest, I point out the curious fact that so profound a mathematician as the late Dean Peacock went quite astray upon the subject (pp. 250-253). Other new sections are those

in which I introduce the idea of negative and approximately zero value, showing that negative value may be brought under the forms of the equations of exchange without any important modification. Readers of Mr. Macleod's works are of course familiar with the idea of negative value; but it was desirable for me to show how important it really is, and how naturally it falls in with the principles of the theory. I may also draw attention to the section (pp. 102-106) in which I illustrate the mathematical character of the equations of exchange by drawing an exact analogy between them and the equations applying to the equilibrium of the lever.

Two or three correspondents, especially Herr Harald Westergaard of Copenhagen, have pointed out that a little manipulation of the symbols, in accordance with the simple rules of the differential calculus, would often give results which I have laboriously argued out. The whole question is one of maxima and minima, the mathematical conditions of which are familiar to mathematicians. But, even if I were capable of presenting the subject in the concise symbolic style satisfactory to the taste of a practised mathematician, I should prefer in an essay of this kind to attain my results by a course of argument which is not only fundamentally true, but is clear and convincing to many readers who, like myself, are not skilful and professional mathematicians. In short, I do not write for mathematicians, nor as a mathematician, but as an economist wishing to convince

other economists that their science can only be satisfactorily treated on an explicitly mathematical basis. When mathematicians recognise the subject as one with which they may usefully deal, I shall gladly resign it into their hands. I have expressed a feeling in more than one place that the whole theory might probably have been put in a more general form by treating labour as a negative utility, and thus bringing it under the ordinary equations of exchange. But the fact is there is endless occupation for an economist in developing and improving his science, and I have found it requisite to reissue this essay, as the bibliopoles say, "with all faults." I have, however, carefully revised every page of the book, and have reason to hope that little or no real error remains in the doctrines stated. The faults are in the form rather than the matter.

Among minor alterations, I may mention the substitution for the name Political Economy of the single convenient term *Economics*. I cannot help thinking that it would be well to discard, as quickly as possible, the old troublesome double-worded name of our Science. Several authors have tried to introduce totally new names, such as Plutology, Chrematistics, Catallactics, etc. But why do we need anything better than Economics? This term, besides being more familiar and closely related to the old term, is perfectly analogous in form to *Mathematics*, *Ethics*, *Æsthetics*, and the names of various other branches of knowledge, and it has moreover the

authority of usage from the time of Aristotle. Mr. Macleod is, so far as I know, the re-introducer of the name in recent years, but it appears to have been adopted also by Mr. Alfred Marshall at Cambridge. It is thus to be hoped that *Economics* will become the recognised name of a science, which nearly a century ago was known to the French Economists as *la science économique*. Though employing the new name in the text, it was obviously undesirable to alter the title-page of the book.

When publishing a new edition of this work, eight years after its first appearance, it seems natural that I should make some remarks upon the changes of opinion about economic science which have taken place in the interval. A remarkable discussion has been lately going on in the reviews and journals concerning the logical method of the science, touching even the question whether there exists any such science at all. Attention was drawn to the matter by Mr. T. E. Cliffe Leslie's remarkable article[1] "On the Philosophical Method of Political Economy," in which he endeavours to dissipate altogether the deductive science of Ricardo. Mr. W. T. Thornton's writings have a somewhat similar tendency. The question has been further stirred up by the admirable criticism to which it was subjected in the masterly address of Professor J. K. Ingram, at the last meeting

[1] *Hermathena*, No. iv., 1876, pp. 1-32. Republished in Mr. Leslie's collected *Essays in Political and Moral Philosophy*, Dublin, 1879, pp. 216-242.

of the British Association. This Address has been reprinted in several publications[1] in England, and has been translated into the chief languages of Western Europe. It is evident, then, that a spirit of very active criticism is spreading, which can hardly fail to overcome in the end the prestige of the false old doctrines. But what is to be put in place of them? At the best it must be allowed that the fall of the old orthodox creed will leave a chaos of diverse opinions. Many would be glad if the supposed science collapsed altogether, and became a matter of history, like astrology, alchemy, and the occult sciences generally. Mr. Cliffe Leslie would not go quite so far as this, but would reconstruct the science in a purely inductive or empirical manner. Either it would then be a congeries of miscellaneous disconnected facts, or else it must fall in as one branch of Mr. Spencer's Sociology. In any case, I hold that *there must arise a science of the development of economic forms and relations.*

But as regards the fate of the deductive method, I disagree altogether with my friend Mr. Leslie; he is in favour of simple deletion; I am for thorough reform and reconstruction. As I have previously explained,[2] the present chaotic state of Economics

[1] *Journal of the London Statistical Society*, December 1878, vol. xli. pp. 602-629. *Journal of the Statistical and Social Inquiry Society of Ireland*, August 1878, vol. vii. Appendix. Also as a separate publication, Longmans, London, 1878.

[2] "The Future of Political Economy," *Fortnightly Review*, November 1876, vol. viii., N.S., pp. 617-631. Translated in the *Journal des Économistes*, March 1877, 3me Série, vol. xlv., p. 325.

arises from the confusing together of several branches of knowledge. Subdivision is the remedy. We must distinguish the empirical element from the abstract theory, from the applied theory, and from the more detailed art of finance and administration. Thus will arise various sciences, such as commercial statistics, the mathematical theory of economics, systematic and descriptive economics, economic sociology, and fiscal science. There may even be a kind of cross subdivision of the sciences; that is to say, there will be division into branches as regards the subject, and division according to the manner of treating the branch of the subject. The manner may be theoretical, empirical, historical, or practical; the subject may be capital and labour, currency, banking, taxation, land tenure, etc.—not to speak of the more fundamental division of the science as it treats of consumption, production, exchange, and distribution of wealth. In fact, the whole subject is so extensive, intricate, and diverse, that it is absurd to suppose it can be treated in any single book or in any single manner. It is no more one science than statics, dynamics, the theory of heat, optics, magneto-electricity, telegraphy, navigation, and photographic chemistry are one science. But as all the physical sciences have their basis more or less obviously in the general principles of mechanics, so all branches and divisions of economic science must be pervaded by certain general principles. It is to the investigation of such principles—to the tracing out of the mechanics

of self-interest and utility, that this essay has been devoted. The establishment of such a theory is a necessary preliminary to any definite drafting of the superstructure of the aggregate science.

Turning now to the theory itself, the question is not so much whether the theory given in this volume is true, but whether there is really any novelty in it. The exclusive importance attributed in England to the Ricardian School of Economists, has prevented almost all English readers from learning the existence of a series of French, as well as a few English, German, or Italian economists, who had from time to time treated the science in a more or less strictly mathematical manner. In the first edition (pp. 14-18), I gave a brief account of such writings of the kind as I was then acquainted with; it is from the works there mentioned, if from any, that I derived the idea of investigating Economics mathematically. To Lardner's *Railway Economy* I was probably most indebted, having been well acquainted with that work since the year 1857. Lardner's book has always struck me as containing a very able investigation, the scientific value of which has not been sufficiently estimated; and in chapter xiii. (pp. 286-296, etc.) we find the Laws of Supply and Demand treated mathematically and illustrated graphically.

In the preface to the first edition (p. xi),[1] I remarked that in his treatise on Political Economy, M. Joseph Garnier mentioned several continental

[1] See p. ix of this edition.

mathematicians who had written on the subject of Economics, and I added that I had not been able to discover even the titles of their memoirs. This, however, must have been the result of careless reading or faulty memory, for it will be found that Garnier himself[1] mentions the titles of several books and memoirs. The fact is that, writing as I did then at a distance from any large library, I made no attempt to acquaint myself with the literature of the subject, little thinking that it was so copious and in some cases so excellent as is now found to be the case. With the progress of years, however, my knowledge of the literature of political economy has been much widened, and the hints of friends and correspondents have made me aware of the existence of many remarkable works which more or less anticipate the views stated in this book. While preparing this new edition, it occurred to me to attempt the discovery of all existing writings of the kind. With this view I drew up a chronological list of all the mathematico-economic works known to me, already about seventy in number, which list, by the kindness of the editor, Mr. Giffen, was printed in the *Journal of the London Statistical Society* for June 1878 (vol. xli. pp. 398-401), separate copies being forwarded to the leading economists, with a request for additions and corrections. My friend, M. Léon Walras, Rector of the Academy of Lausanne, after himself making considerable additions to the list, communicated it to

[1] *Traité d'Économie Politique*, 5me éd., Paris, 1863, pp. 700-2.

the *Journal des Économistes* (December 1878), to the editor of which we are much indebted for its publication. Copies of the list were also sent to German and Italian economical journals. For the completion of the bibliographical list I am under obligations to Professor W. B. Hodgson, Professor Adamson, Mr. W. H. Brewer, M.A., H.M. Inspector of Schools, the Baron d'Aulnis de Bourouill, Professor of Political Economy at Utrecht, M. N. G. Pierson of Amsterdam, M. Vissering of Leiden, Professor Luigi Cossa of Pavia, and others.

All reasonable exertions have thus been made to render complete and exhaustive the list of mathematico-economic works and papers, which is now printed in the first [now fifth] Appendix to this book (pp. 322-339). It is hardly likely that many additions can be made to the earlier parts of the lists, but I shall be much obliged to any readers who can suggest corrections or additions. I shall also be glad to be informed of any new publications suitable for insertion in the list. On the other hand, it is possible that some of the books mentioned in the list ought not to be there. I have not been able in all cases to examine the publications myself, so that some works inserted at the suggestion of correspondents may have been named under misconception of the precise purpose of the list. Economic works, for instance, containing numerical illustrations and statistical facts numerically expressed, however abundantly, have not been intentionally included, unless there was also mathematical method in the reasoning. Without this condition the whole literature of numerical commercial statistics would have been imported into my list. In other cases only a small portion of a book named can be called mathematico-economic; but this fact is generally noted by the quotation of the chapters or pages in question. The tendency, however, has been to include rather than to exclude, so that the reader might have before him the whole field of literature requiring investigation.

To avoid misapprehension it may be well to explain that the ground for inserting any publication or part of a publication in this list, is its containing *an explicit recognition of the mathematical character of economics, or the advantage to be attained by its symbolical treatment.* I contend that all economic writers must be mathematical so far as they are scientific at all, because they treat of economic quantities, and the relations of such quantities, and all quantities and relations of quantities come within the scope of the mathematics. Even those who have most strongly and clearly protested against the recognition of their own method, continually betray in their language the quantitative character of their reasonings. What, for instance, can be more clearly mathematical in matter than the following quotation from Cairnes's chief work:[1]—"We can have no difficulty in seeing how cost in its principal

[1] *Some Leading Principles of Political Economy Newly Expounded*, pt. 1, chap. iii. p. 97.

exceeds the desire of possession, including all the motives for exertion.

We must consider the duration of labour as measured by the number of hours' work per day. The alternation of day and night on the earth has rendered man essentially periodic in his habits and actions. In a natural and wholesome condition a man should return each twenty-four hours to exactly the same state; at any rate, the cycle should be closed within the seven days of the week. Thus the labourer must not be supposed to be either increasing or diminishing his normal strength. But the theory might also be made to apply to cases where special exertion is undergone for many days or weeks in succession, in order to complete work, as in collecting the harvest. Adequate motives may lead to and warrant overwork, but, if long continued, excessive labour reduces the strength and becomes insupportable; and the longer it continues the worse it is, the law being somewhat similar to that of periodic labour.

Symbolic Statement of the Theory

In attempting to represent these conditions of labour with accuracy, we shall find that there are no less than four quantities concerned; let us denote them as follows:—

t = time, or duration of labour;

l = amount of labour, as meaning the aggregate balance of pain accompanying it, irrespective of the produce;

x = amount of commodity produced;

u = total utility of that commodity.

The amount of commodity produced will be very different in different cases. In any one case the rate of production will be determined by dividing the whole quantity produced by the time of production, provided that the rate of production has been uniform; it will then be $\frac{x}{t}$. But if the rate of production be variable, it can only be determined at any moment by comparing a small quantity of produce with the small portion of time occupied in its production. More strictly speaking, we must ascertain the ratio of an infinitely small quantity of produce to the corresponding infinitely small portion of time. Thus *the rate of production* is properly denoted by $\frac{\Delta x}{\Delta t}$, or at the limit by $\frac{dx}{dt}$.

Again, the degree of painfulness of labour would be $\frac{l}{t}$ if it remained invariable; but as it is highly variable, we must again compare small increments, and $\frac{\Delta l}{\Delta t}$, or, at the limit, $\frac{dl}{dt}$ correctly represents the *degree of painfulness of labour*. But we must also take into account the fact that the utility of commodity is not constant. If a man works regularly twelve hours a day, he will produce more commodity than in ten hours; therefore the final degree of utility of his commodity, whether he consume it himself or